Prep 1

Second term

لطلب المذكرة ببياناتك تواصل واتس / 01032243340

Name / ....

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Unit 1, Lesson 1 | Exponents and Powers

learn

- $3^4 = 3 \times 3 \times 3 \times 3$ we read it as : «3 to the power 4»

  For example: $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

  $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2^4}{3^4} = \left(\frac{2}{3}\right)^4$

- If $\frac{a}{b}$ is a rational number and n is a positive integer then :

  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, For example: $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$

- If $\frac{a}{b}$ is a rational number, then :

  $\left(\frac{a}{b}\right)^0 = 1$ where $a \neq 0$, For example: $\left(\frac{1}{5}\right)^0 = 1$

- If a is a rational number and m is a positive integer

  when m is an even number then : $(-\mathbf{a})^m = (\mathbf{a})^m$, For example: $\left(-\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

  when m is an odd number then : $(-a)^m = -(a)^m$, For example: $\left(-\frac{1}{2}\right)^3 = -\left(\frac{1}{2}\right)^3 = -\frac{1}{8}$

Example: Write the following in exponential form so that the base is a prime number:

1	$125 = 5 \times 5 \times 5 = 5^3$	2	$108 = 4 \times 27 = 2^2 \times 3^3$

Example 1 :Find each of the following in the simplest form:

1	$\left(\frac{2}{3}\right)^2 \times \frac{9}{4} =$ $= \frac{2^2}{3^2} \times \frac{9}{4} = \frac{4}{9} \times \frac{9}{4} = 1$	2	$\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{5}\right)^4$ $= \frac{5^2}{4^2} \times \frac{2^4}{5^4} = \frac{25}{16} \times \frac{16}{625} = \frac{1}{25}$
3	$\left(3\frac{1}{2}\right)^2 \div \left(-10\frac{1}{2}\right) =$ $= \left(\frac{7}{2}\right)^2 \div \left(-\frac{21}{2}\right)$ $= \frac{7^2}{2^2} \times \left(-\frac{2}{21}\right) = \frac{49}{4} \times \left(-\frac{2}{21}\right) = -\frac{7}{6}$	4	$\left(-\frac{2}{5}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0 =$ $= \frac{2^2}{5^2} \times \left(-\frac{5^3}{2^3}\right) \times 1$ $= \frac{4}{25} \times \left(-\frac{125}{8}\right) = -\frac{5}{2}$
5	$\left(\frac{1}{4}\right)^2$	6	$\left(-\frac{5}{3}\right)^3$ ....

7	$\left(-\frac{4}{7}\right)^4$	8	$\left(2\frac{1}{2}\right)^4$ ....
9	$\left(-\frac{3}{9}\right)^2 \times \left(\frac{9}{4}\right)^2 \times \left(\frac{81}{16}\right)^0$	10	$\left(-\frac{1}{4}\right)^3 \times \left(\frac{2}{3}\right)^2$ ....

Example 2

1	**If $x = -\frac{1}{2}, y = \frac{1}{4}$ and $z = 4$, find the value of : $(x+y)^3 \times z^3$** **solu** $(x+y)^3 \times z^3 = \left(-\frac{1}{2}+\frac{1}{4}\right)^3 \times 4^3 = \left(-\frac{2}{4}+\frac{1}{4}\right)^3 \times 4^3$ $= \left(-\frac{1}{4}\right)^3 \times 4^3 = -\frac{1^3}{4^3} \times 4^3 = -1$
2	**If $x = -\frac{2}{3}, y = \frac{1}{2}$ and $z = -\frac{4}{3}$, find the value of: $x^2 - y^2 z$** ....
3	$\sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^{zero} \times \left(\frac{-2}{7}\right)^2 =$ ....
4	$\frac{2}{5} \times \sqrt{\frac{9}{16}} \div \left(-\frac{1}{2}\right)^3 =$ ....
5	$\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{4}\right)^{zero} =$ ....

learn

- If $\frac{a}{b}$ is a rational number, n and m are non-negative integers ,then
  $\left(\frac{a}{b}\right)^{n} \times \left(\frac{a}{b}\right)^{m} = \left(\frac{a}{b}\right)^{n+m}$, For example: $\left(\frac{2}{5}\right)^{3} \times \left(\frac{2}{5}\right)^{2} = \left(\frac{2}{5}\right)^{3+2} = \left(\frac{2}{5}\right)^{5}$
- If $\frac{a}{b}$ is a rational number, where $\frac{a}{b} \neq 0$, n and m are non-negative integers, $n \geq m$,
  then: $\left(\frac{a}{b}\right)^{n} \div \left(\frac{a}{b}\right)^{m} = \left(\frac{a}{b}\right)^{n-m}$, For example: $\left(\frac{3}{8}\right)^{5} \div \left(\frac{3}{8}\right)^{2} = \left(\frac{3}{8}\right)^{5-2} = \left(\frac{3}{8}\right)^{3}$
- If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, n is a non-negative integer, then :
  $\left(\frac{a}{b} \times \frac{c}{d}\right)^{n} = \left(\frac{a}{b}\right)^{n} \times \left(\frac{c}{d}\right)^{n}$ For example: $\left(\frac{3}{4} \times \frac{5}{7}\right)^{3} = \left(\frac{3}{4}\right)^{3} \times \left(\frac{5}{7}\right)^{3}$
- If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , $\frac{c}{d} \neq 0$, n is a non-negative integer , then :
  $\left(\frac{a}{b} \div \frac{c}{d}\right)^{n} = \left(\frac{a}{b}\right)^{n} \div \left(\frac{c}{d}\right)^{n}$ (where $\frac{c}{d} \neq 0$)
- If $\frac{a}{b}$ is a rational number, n and m are non-negative integers
  , then : $\left[\left(\frac{a}{b}\right)^{n}\right]^{m} = \left(\frac{a}{b}\right)^{n \times m}$ For example: $\left[\left(\frac{3}{5}\right)^{3}\right]^{2} = \left(\frac{3}{5}\right)^{3\times2} = \left(\frac{3}{5}\right)^{6}$

Example 1 Find each of the following in the simplest form.

1	$\frac{2}{3} \times \left(\frac{2}{3}\right)^{2} \times \left(\frac{2}{3}\right)^{3} =$ $= \left(\frac{2}{3}\right)^{1+2+3} = \left(\frac{2}{3}\right)^{6} = \frac{2^6}{3^6} = \frac{64}{729}$	2	$\left(-\frac{1}{3}\right)^{3} \times \left(\frac{1}{3}\right)^{2} =$ $-\left(\frac{1}{3}\right)^{3} \times \left(\frac{1}{3}\right)^{2} = -\left(\frac{1}{3}\right)^{5} = -\frac{1^5}{3^5} = -\frac{1}{243}$
3	$\left(-\frac{2}{7}\right)^{4} \div \left(-\frac{2}{7}\right)^{2} =$ $\left(-\frac{2}{7}\right)^{4-2} = \left(-\frac{2}{7}\right)^{2}$	4	$\frac{3}{4} \times \left(-\frac{3}{4}\right)^{2}$ ....
5	$\frac{2^5 \times 2^4}{2^6}$	6	$\left(\frac{4}{5}\right)^{2} \times \left(\frac{4}{5}\right)^{5} \div \left(\frac{4}{5}\right)^{7}$ ....

7	$\left(\frac{3}{7}\right)^8 \div \left(\frac{3}{7}\right)^6$	8	$\left(\frac{1}{5}\right)^2 \times \left(\frac{1}{5}\right)^2$ ....
9	$\left(-\frac{1}{4}\right)^7 \div \left(\frac{1}{4}\right)^6 \times \frac{1}{4}$	10	$\left(-\frac{2}{3}\right)^5 \times \left(-\frac{2}{3}\right)^2 \div \left(-\frac{2}{3}\right)^6$ ....
11	$\left[\left(-2\frac{1}{2}\right)^2\right]^2$	12	$\left(\frac{2x}{3y}\right)^4 =$ ....
13	$\frac{(-4x^3y^4)^2}{(-2xy^2)^4} =$	14	$\left(\frac{x^2}{y^3}\right)^3 =$ ....
15	$\left(\frac{5^2\times5^4}{5^5}\right)^2 =$	16	$\left(\frac{a^2b^2}{c^3d^4}\right)^2 =$ ....
17	$\frac{7^8\times7^3\times7}{7^{10}} =$	18	$\frac{(-4)^2\times4^8}{(-4)\times(-4)^6} =$ ....

Example 4: Find the numerical value of each of the following quantities at the given values.

1	b = 5 , a = 2 , $(2a)^b$ = ...
2	b = -3 , a = 3 , $(-2b)^a$ = ...
3	b = $\frac{2}{3}$, a = 4 , $(b)^a$ = ...
4	b = $\frac{-1}{5}$, a = 3 , $(-3b)^a$ = ...

Exercises (1)

Example: Write the following in exponential form so that the base is a prime number:

1	$625 =$	2	$72 =$ ....
3	$\frac{1}{16} =$	4	$\frac{1}{36} =$ ....

[1] Calculate each of the following, then put the resutl in the simplest form:

1	$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2 =$	2	$\left(\frac{-2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2 =$ ....
3	$\left(\frac{1}{5}\right) \times \left(\frac{-1}{5}\right)^4 =$	4	$\left(\frac{1}{6}\right)^9 \div \left(\frac{1}{6}\right)^8 =$ ....
5	$\left(\frac{2}{7}\right)^5 \div \left(\frac{2}{7}\right)^3 =$	6	$\left(\frac{-3}{5}\right)^7 \div \left(\frac{3}{5}\right)^5 =$ ....
7	[illegible]	8	[illegible] ....

[2] Calculate each of the following, then put the result in the simplest form:

1	$\frac{3^7 \times 3^3}{3^6} =$	2	$\frac{2^6 \times 2}{2^3 \times 2^4} =$ ....
3	$\frac{(-5)^4 \times 5^2}{5^3} =$	4	$\frac{(-2)^5 \times 2^4}{(-2)^3 \times 2^2} =$ ....
5	$\frac{(-3)^5 \times (-2)^7}{(-3)^3 \times (-2)^5} =$	6	$\frac{x^2 \times x^3 \times x^4}{x^7 \times x} =$ ....
7	$\frac{x^4 \times y^3 \times x^5}{x^6 \times y^2} =$	8	$\left(\frac{ab}{c}\right)^5 =$ ....
9	$\left(\frac{5x}{3y}\right)^2 =$	10	$\left(\frac{-2ab}{3c}\right)^4 =$ ....

11	$\left(\frac{x^2}{y^3}\right)^2 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$	12	$\left(\frac{a^3b^2}{c^5}\right)^3 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
13	$\left(\frac{-c^2}{d}\right)^3 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$	14	$\left(\frac{-x^3}{y^2}\right)^2 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ ....
15	$\left[\left(\frac{1}{2}\right)^2\right]^2 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$	16	$\left[\left(\frac{-3}{2}\right)^2\right]^5 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ ....
17	$\left[\left(2\frac{1}{2}\right)^3\right]^2 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$	18	$\left(\frac{3}{5}\right)^{10} \times \left(\frac{5}{3}\right)^{10} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ ....
19	$\left[\left(\frac{2}{7}\right)^2\right]^3 \times \left(\frac{7}{2}\right)^6 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$	20	$\left(2\frac{1}{2}\right)^2 \times \left(\frac{-2}{5}\right)^2 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ ....

[3] Complete:

1	$\frac{9}{16} = \left(\frac{3}{4}\right)^{\cdots}$	2	$\frac{8}{27} = \left(\frac{2}{3}\right)^{\cdots}$
3	$64\% = \left(\frac{4}{5}\right)^{\cdots}$	4	$0.027 = \left(\frac{3}{10}\right)^{\cdots}$
5	$2^2 + 2^2 = 2^{\cdots}$	6	If $\frac{x}{y} = \frac{-2}{5}$, then $\left(\frac{x}{y}\right)^3 = \ldots\ldots$
7	$\left(\frac{1}{2}\right)^3 = \ldots\ldots\ldots$	8	$\left(\frac{-1}{2}\right)^3 - \left(\frac{-1}{2}\right)^2 = \ldots\ldots\ldots\ldots$
9	$\left(\frac{1}{3}\right)^4 = \ldots\ldots\ldots$	10	$\left(\frac{-1}{7}\right)^3 = \ldots\ldots\ldots\ldots$
11	The additive inverse of the number $(-3)^0$ is ……….	12	The multiplicative inverse of the number $\left(\frac{2}{5}\right)^0 = \ldots\ldots$
13	$\left(\frac{a}{b}\right)^2 \times \frac{b^2}{a^2} = \cdots\ldots\ldots$ (where $ab \neq 0$)	14	If $x = y$, then $\left(\frac{3}{5}\right)^{x-y} = \cdots$

Example 4: Find the numerical value of each of the following quantities at the given values.	
1	$b = 2$, $a = 3$, $(4a)^b = \ldots\ldots\ldots\ldots\ldots\ldots$
2	$b = -2$, $a = 2$, $(-3b)^a = \ldots\ldots\ldots\ldots\ldots$
3	$b = \frac{2}{5}$, $a = 2$, $(b)^a = \ldots\ldots\ldots\ldots\ldots\ldots$

Unit 1, Lesson 1 | Exponents and Powers

learn

- If a is a rational number , $a \neq 0$ and n is a positive integer, then

  $a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$, For example: $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

  Remarks

  (1) If a is a rational number, $a \neq 0$ and n is a positive integer, then

  $a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$ (the multiplicative neutral)

  (2) If $\frac{a}{b}$ is a rational number not equal to zero and n is a positive integer , then :

  $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$, For example: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Example 1 Find the value of each of the following in the simplest form :

1	$\frac{5^{-2}}{5^{-3}} = \frac{5^3}{5^2} = 5^{3-2} = 5$	2	$12^4 \times 2^{-2} =$ $2^4 \times \frac{1}{2^2} = \frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4$
3	$\frac{6^{-3} \times 6^{-5}}{6^2} = \frac{6^5}{6^3 \times 6^2} = \frac{6^5}{6^5} = 1$	4	$(3^2)^{-2} = \frac{1}{(3^2)^2} = \frac{1}{3^4} = \frac{1}{81}$
5	$(7^3)^2 \times (7^{-2})^2 = (7^3)^2 \times \left(\frac{1}{7^2}\right)^2$ $= 7^6 \times \frac{1}{7^4} = 7^{6-4} = 7^2 = 49$	6	$\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2} = \left(\frac{5^3 \times 5}{5^2 \times 5^4}\right)^{-2} = \left(\frac{5^4}{5^6}\right)^{-2}$ $\left(\frac{5^6}{5^4}\right)^2 = (5^{6-4})^2 = (5^2)^2 = 5^4 = 625$
7	$\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2} =$ $(5^{3+(-2)-(-1)-4})^{-2} = (5^{3-2+1-4})^{-2}$ $= (5^{-2})^{-2} = 5^{(-2)\times(-2)} = 5^4 = 625$	8	$\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3} =$ $\left(\frac{5}{3}\right)^3 \div \left(\frac{5}{4}\right)^3 = \left(\frac{5}{3} \div \frac{5}{4}\right)^3$ $= \left(\frac{5}{3} \times \frac{4}{5}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$
9	$(2^{-3})^2 =$	10	$\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3} =$ ....
11	$\left(\frac{2^{-2} \times 2^6}{2^3}\right)^{-3} =$	12	$\left(\frac{3}{7}\right)^{-2} =$ ....

13	$\frac{a^{-1}\times a^{2}\times a^{-3}}{a^{4}\times a^{-7}}$ = ...	14	$\frac{-3\times 5^{-3}\times 2^{5}}{2^{3}\times 3^{-1}\times 5^{-4}}$ =............................ ....

Example 2 Simplify each of the following to the simplest form where $\chi \neq 0$	
1	$x^5 \times x^{-2} \times x^{-3} = x^{5+(-2)+(-3)} = x^{5-2-3} = x^0 = 1$
2	$(x^2)^{-3} \div (x^{-1})^2 = x^{-6} \div x^{-2} = x^{-6-(-2)} = x^{-6+2} = x^{-4} = \frac{1}{x^4}$
3	$\left(\frac{x^4\times x^{-3}}{x^{-4}\times x}\right)^{-2} = \left(x^{4+(-3)-(-4)-1}\right)^{-2} = (x^{4-3+4-1})^{-2} = (x^4)^{-2} = x^{-8} = \frac{1}{x^8}$
4	$(x^{-2})^{-5}$ = ...
5	$\left(\frac{a^4}{a^{-3}}\right)^{-2}$ = ...
6	$(y^5 \times y^{-2})^3$ = ...

Example 3	
1	If you know that the mass of the Sun is approximately 10^{27} tons, what would the mass of the Sun be in kilograms? (Write the result in scientific notation with base 10) ....
2	**Without calculating the values, find the median of the following numbers** 2^{-4} ، 2^{3} ، 2^{-3} ، 2^{0} ، 2^{-2} ....

Exercises (2)

[illegible]

1. $4^{-1} = \dots\dots\dots$
2. $5^{-2} = \dots\dots\dots$
3. $\left(\frac{1}{2}\right)^{-1} = \dots\dots\dots$
4. $\left(-\frac{2}{3}\right)^{-2} = \dots\dots\dots$
5. $(0.2)^{-2} = \dots\dots\dots$
6. $(1.2)^{-1} = \dots\dots\dots$
7. If $a = 7^{x}, b = 7^{-x}$, then $a \times b = \cdots$
8. If $a = \frac{2}{3}$, then $a^{-1} = \dots\dots\dots$

[illegible]

1. $3^{7} \times 3^{-3} = \dots\dots\dots$
2. $2^{-2} \times 2^{-3} = \dots\dots\dots$
3. $\frac{3}{3^{-2}} = \dots\dots\dots$
4. $\frac{6^{-2}}{6^{-3}} = \dots\dots\dots$

[illegible]

1. $\frac{8 \times 8^{-2}}{8^{-3}} = \dots\dots\dots$
2. $\frac{7^{-2} \times 7^{5}}{7^{3}} = \dots\dots\dots$
3. $\frac{2^{5} \times 2^{-2}}{2^{-4} \times 2^{3}} = \dots\dots\dots$
4. $(5^{-1})^{-3} = \dots\dots\dots$
5. $(3^{-2})^{2} = \dots\dots\dots$
6. $(0.25)^{-2} = \dots\dots\dots$
7. $(2^{-1} \times 2^{-2})^{3} = \dots\dots\dots$
8. $\left(\frac{3^{-1}}{3}\right)^{2} = \dots\dots\dots$

$\left(\frac{8^4}{8^{-4}}\right)^0 =$ ....

....

....

$\frac{(3^{-2})^3}{3^{-2}\times 3^{-6}} =$ ....

....

....

$7x^{-1} =$ ....

....

$x^{-1}y^2 =$ ....

....

$a^{-2}b^{-3} =$ ....

....

$x^3 \times x^{-5} =$ ....

....

$x^3 \times x^{-2} \times x^{-1} =$ ....

....

$\frac{c^{-5}}{c^2} =$ ....

....

$x^7 \div x^{-5} =$ ....

....

$(a^{-2})^3 =$ ....

....

$(x^2)^{-3} \times (x^{-3})^{-2} =$ ....

....

$(b^{-1})^{-3}$

....

$2x^{-3} = \frac{2}{\dots\dots}$

$(3x^{-1})^2 = 9x^{\dots} = \frac{9}{\dots\dots}$

$(3a^2)^{-1} = \frac{1}{\dots\dots}$

$2x^{-2}y^{-3} = \frac{2}{\dots\dots}$

$3^2 \times 3^5 =$			
3^{25}	3^{10}	3^3	3^7
$5^2 + 5^2 =$			
50	5^4	10^4	10^2
$3^5 \times 2^5 =$			
6^{25}	6^5	6^{10}	5^{10}
$(5a)^0 = \cdots$, $a \neq 0$			
1	$5a$	a	5
$3^{(2^3)} = \dots$			
3^{23}	3^8	3^5	3^6
$(5^2)^3 = \dots$			
5	5^{23}	5^5	5^6

$3^{10}+3^{10}+3^{10}=$ .....

(a) 9^{10} (b) 3^{11} (c) 3^{30} (d) 3^{10}

$4^x+4^x+4^x+4^x=$

(a) $4x^4$ (b) 4^{x+1} (c) 4^{4x} (d) 4^{x+4}

$\frac{(3^2)^5}{(3^5)^2}=$

(a) 1 (b) 3^{25} (c) 3^{52} (d) 3^{10}

$(2y)^3=$ ....

(a) $23y$ (b) $8y^3$ (c) $8y$ (d) $2y^3$

$(b^3)^4=$ ..

(a) $b^4\times b^4\times b^4$ (b) b^7 (c) $b^3\times b^3\times b^3$ (d) b^{34}

The quarter of the number 4^{20} is .....

(a) 2^{10} (b) 4^{19} (c) 4^{10} (d) 4^5

If $a^{-1}=\frac{2}{3}$, then $a=\cdots$

(a) 1 (b) $-\frac{3}{2}$ (c) $\frac{3}{2}$ (d) $\frac{-2}{3}$

If $a=7^x$ and $b=7^{-x}$, then $a\times b=\cdots$

(a) 0 (b) 1 (c) 49^{2x} (d) 7^{2x}

$\frac{6a^2x^4}{2a^3x^3}=\cdots$

(a) $\frac{3}{ax}$ (b) $\frac{3x}{a}$ (c) $3a^5x^7$ (d) $3ax$

$\frac{(-2s^2t)^3}{(-4st^2)^2}=\cdots$

(a) $\frac{s^4}{t}$ (b) $\frac{s^5}{2t^2}$ (c) $\frac{-s^4}{2t}$ (d) $\frac{-s^3}{2t}$

If $a^x=2$ and $a^{-y}=3$, then $a^{x-y}=\cdots$

(a) 6 (b) $\frac{2}{3}$ (c) -1 (d) 1

If $xy^{-1}=\frac{1}{2}$, then $\frac{y}{x}=\ldots$

(a) 2 (b) 1 (c) $\frac{-1}{2}$ (d) $\frac{1}{2}$

$3^{-1}+3^{-1}+3^{-1}=$

(a) 1 (b) 9^{-3} (c) 3^3 (d) 3^{-3}

The multiplicative inverse of 5^{-1} is .....

(a) $\frac{-1}{5}$ (b) -5 (c) 5 (d) $\frac{1}{5}$

The multiplicative inverse of 5^{-1} is .....

(a) 0 (b) $\left(\frac{3}{5}\right)^{-4}$ (c) 1 (d) $\left(\frac{3}{5}\right)^{4}$

Unit 1, Lesson 2 | The scientific notation of the number

learn

- **The standard scientific notation of a number**

  The number is written in the standard form as: $a \times 10^n$ where $1 \le |a| < 10$ and $n \in \mathbb{Z}$

  examples : 4.6×10^8 , 5.236×10^{-6} , -9.6×10^{10} , 1×10^{-7}

  Remark : The standard form for 1 is 1×10^{zero}

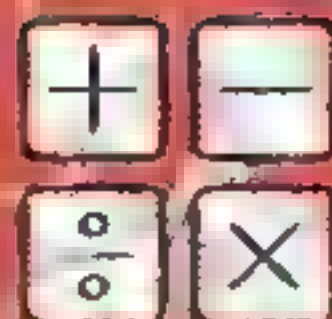
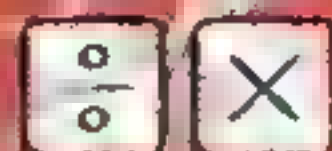
- Examples for some numbers not in the standard form :

  45×10^8 (because $45 > 10$)

  706.4×10^5 (because $706.4 > 10$)

  0.248×10^{-7} (because $0.248 < 1$)

[illegible]

$0.000000135 = 1.35 \times 10$ (-7) Moving the decimal point 7 places towards right	$1,820,000,000 = 1.82 \times 10^9$ ($+9$) Moving the decimal point 9 places towards left
$0.248 \times 10^{-7} =$ $2.48 \times 10^{-7} \times 10^{-1} = 2.48 \times 10^{-8}$ $-0.0015 \times 10^{-9} =$	$706.4 \times 10^5 =$ $7.064 \times 10^5 \times 10^2 = 7.064 \times 10^7$ $45.0 \times 10^8 =$ ....
$0.5 \times 10^{-7} =$	$17 \times 10^8 =$ ....
650000000 = ...	$80.012 =$ ....
530.5×10^9	-0.999×10^{-5} ....

1	$(1.2 \times 10^5) \times (4 \times 10^3) = (1.2 \times 4) \times (10^5 \times 10^3) = 4.8 \times 10^8$
2	$(6.5 \times 10^4) \times (8 \times 10^2) = (6.5 \times 8) \times (10^4 \times 10^2) = 52 \times 10^6 = 5.2 \times 10^7$
3	$(2.4 \times 10^{11}) \div (1.2 \times 10^{-4}) = \frac{2.4}{1.2} \times \frac{10^{11}}{10^{-4}} = 2 \times 10^{15}$
4	$(6.6 \times 10^7) \times (3 \times 10)^4 = (6.6 \times 10^7) \times (3^4 \times 10^4)$ $= (6.6 \times 3^4) \times (10^7 \times 10^4)$ $= 534.6 \times 10^{11} = 5.346 \times 10^{13}$
5	$(2.3 \times 10^6) + (3.7 \times 10^5) = 10^5(2.3 \times 10 + 3.7)$ $= 10^5(23 + 3.7) = 10^5 \times 26.7 = 2.67 \times 10^6$
6	$30000 \times 400000 = (3 \times 10^4) \times (4 \times 10^5) = (3 \times 4) \times (10^4 \times 10^5)$ $= 12 \times 10^9 = 1.2 \times 10^{10}$
7	$0.000015 \div 30 = (1.5 \times 10^{-5}) \div 3 \times 10 = \frac{1.5}{3} \times \frac{10^{-5}}{10} = 0.5 \times 10^{-6} = 5 \times 10^{-7}$
8	$(50000)^3 = (5 \times 10^4)^3 = 5^3 \times 10^{12} = 125 \times 10^{12} = 1.25 \times 10^{14}$
9	$(0.0003)^3 =$ ....
10	$(-0.001)^6 =$ ....
11	$(5.3 \times 10^7) \times (3 \times 10^5) =$ ....
12	$0.0006 \div 20 =$ ....
13	$(400000)^2 =$ ....
14	$(3.2 \times 10^9) - (0.2 \times 10^8) =$ ....
15	$(1.4 \times 10^4) \times (2 \times 10^3) =$ ....

Exercises (3)

[illegible]

1	5.3×10^7	2	0.2×10^{-4} ....
3	0.025×10^8	4	7×10^{-4} ....
5	4.25×10	6	10×10^{-10} ....
7	33.9×10^6	8	-5.783×10^2 ....
9	-0.0003×10^3	10	3.912×10^{-2} ....

[illegible]

1	19 million = ...	2	71×10^5 = ...
3	[illegible]	4	[illegible]
5	$-200,000$ = ...	6	0.0006 = ...
7	0.000053 = ...	8	0.000864 = ...
9	0.421 = ...	10	68×10^5 = ...
11	68×10^{-5} = ...	12	720×10^6 = ...
13	750×10^{-9} = ...	14	-32.4×10^4 = ...
15	0.005×10^{15} = ...	16	0.0020205×10^{12} = ...

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1. $(6.4 \times 10^{8}) \times (1.5 \times 10^{5}) =$	2. $(8.2 \times 10^{7}) \times (2.1 \times 10^{-4}) =$ ....
$(5.02 \times 10^{-4}) \times (0.1 \times 10^{-3}) =$	$(3.8 \times 10^{8}) \div (1.9 \times 10^{6}) =$ ....
$(125.5 \times 10^{-3}) \div (5 \times 10^{4}) =$	$(3.8 \times 10^{5}) + (4.6 \times 10^{4}) =$ ....
$(4.54 \times 10^{4}) + (3.76 \times 10^{3}) =$	$(5.3 \times 10^{8}) - (0.8 \times 10^{7}) =$ ....

$3.04 \times 10^{[illegible]} =$

(a)	(b)	(c)	(d)
30400000	3400000	304000	340000

$2.37 \times 10^{-4} =$

(a)	(b)	(c)	(d)
0.0000237	23700	0.000237	0.00237

If $0.00079 = 7.9 \times a$, then $a =$

(a)	(b)	(c)	(d)
10^{4}	10^{-4}	10^{-3}	10^{3}

If $0.00000503 = m \times 10^{-5}$, then $m =$

(a)	(b)	(c)	(d)
0.503	5.03	50.3	503

$0.0000006 = 6 \times 10^{n}$ then n = ...	$800{,}000 = 8 \times 10^{n}$ then n = ...
$0.000357 = 3.57 \times 10^{n}$ then n = ...	$0.00052 = 5.2 \times 10^{n}$ then n = ...

Unit 1, Lesson 3 | The square root

learn

- **Definition**
  The square root of the perfect square rational number "a" is the number whose square equals " a "
- For example: The number 5 is a square root of the number 25 because : $5^2 = 25$
- Remarks
  (1) $\sqrt{0} = 0$
  (2) In the set of rational numbers it is meaningless to find $\sqrt{a}$ if a is a negative rational number because there is no rational number if it is multiplied by itself, the result will be negative.
  (3) $\sqrt{a^2} = |a|$, For example: $\sqrt{(-2)^2} = |-2| = 2$
  (4) $\sqrt{a^2\,b^2} = \sqrt{(ab)^2} = |ab|$, For example: $\sqrt{a^4\,b^6} = \sqrt{(a^2\,b^3)^2} = |a^2\,b^3|$
  (5) If $x^2 = a$ where $a \geq 0$, then $x = \pm\sqrt{a}$

	$\sqrt{36} =$		$-\sqrt{0.25} =$ ....
	$\pm\sqrt{\frac{3.6}{10}} =$		$\pm\sqrt{2\frac{1}{4}} =$ ....
	$\sqrt{100-36} =$		$\sqrt{16+9} =$ ....
	$\pm\sqrt{6\frac{1}{4}} =$	8	$\sqrt{\frac{36a^8}{49\,d^4}} =$ ....
	$-\sqrt{900} =$	10	$\sqrt{64} =$ ....
	$\sqrt{100-64} =$	12	$\sqrt{0.64} =$ ....

1 $\left(2\frac{7}{9}\right)^2 \div \sqrt{\frac{25}{9}} =$ ....

....

2 $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2 =$ ....

3 $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0 =$ ....

4 $\left(\frac{2}{3}\right)^2 \times \sqrt{\frac{81}{16}} \times \left(\frac{7}{9}\right)^0 =$ ....

Example 4

1 The base length of a triangle is 16 cm . and its corresponding height is 8 cm . Find the side length of a square having the same area of that triangle.

....

....

....

2 The area of a square is 1.44 cm . Find its perimeter.

....

....

....

[illegible]

1 $x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

2 $4x^2 + 4 = 68$

....

....

....

3 $3x^2 - 5 = 43$

....

....

....

Exercises (4)

(1) [illegible]

1. $\sqrt{16} =$ ....
2. $-\sqrt{25} =$ ....
3. $\pm\sqrt{40000} =$ ....
4. $\pm\sqrt{2500} =$ ....
5. $\sqrt{6\frac{1}{4}} =$ ....
6. $\sqrt{\frac{9}{49}} =$ ....
7. $\pm\sqrt{8^2} =$ ....
8. $-\sqrt{4^2} =$ ....
9. $\sqrt{\left(-\frac{3}{4}\right)^2} =$ ....
10. $\sqrt{\left(\frac{81}{100}\right)^2} =$ ....
11. $\pm\sqrt{\frac{16b^8}{121h^2}} =$ ....
12. $\sqrt{\frac{49a^4b^2}{9}} =$ ....
13. $\sqrt{\frac{25x^2y^2}{36}} =$ ....
14. $\sqrt{0.36} =$ ....

(2) [illegible]

1. $\sqrt{9} + \sqrt{16} =$ ....
2. $\sqrt{36 + 64} =$ ....
3. $\sqrt{\frac{5^4 \times 5^3}{5^5}} =$ ....
   ....
4. $\sqrt{3^2 + 4^2} =$ ....
   ....
5. $\sqrt{9} + \sqrt{16} =$ ....
   ....
6. $\sqrt{\frac{9}{16} + 1} =$ ....
   ....
7. $\sqrt{25 - 9} =$ ....
   ....
8. $\sqrt{\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{3}\right)^4} =$ ....
   ....
9. $\frac{3}{4} \times \sqrt{\frac{16}{9}} =$ ....
   ....
10. $\sqrt{\frac{81}{49}} \times \frac{14}{27} =$ ....
    ....
11. $\sqrt{\frac{9}{4}} - \frac{3}{2} + \left(\frac{3}{2}\right)^{zero} =$ ....
    ....
12. $\sqrt{100 - 36} =$ ....
    ....

1. The multiplicative inverse of the number $\sqrt{\frac{4}{25}}$ in the simplest form is ....
2. The multiplicative inverse of the number $\sqrt{0.49}$ in the simplest form is ....
3. The multiplicative inverse of the rational number $\sqrt{\frac{10}{2.5}}$ in the simplest form is ....
4. The additive inverse of the number $-\sqrt{\frac{9}{16}}$ in the simplest form is ....
5. The multiplicative inverse of the number $\sqrt{\frac{9}{16}}$ in the simplest form is ....

[4] Simplify:

1. $\left(\frac{3}{4}\right)^{zero} \times \sqrt{\frac{81}{64}} \times \left(\frac{-2}{3}\right)^3 =$ ....

   ....
2. $\left(\frac{-2}{5}\right)^2 \times \left(\frac{-3}{5}\right)^0 \times \sqrt{6\frac{1}{4}} =$ ....

3. $\sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^{zero} \times \left(\frac{-2}{7}\right)^2 =$ ....

   ....
4. $\frac{2}{5} \times \sqrt{\frac{9}{16}} \div \left(-\frac{1}{2}\right)^3 =$ ....

   ....
5. $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{4}\right)^{zero} =$ ....

   ....

Example 5: Find the value of x.

1. $x^2 - 1 = 8$

   ....

   ....

   ....
2. $2x^2 - 5 = 13$

   ....

   ....

   ....

$\sqrt{1\frac{9}{16}} = ...$

(a) $-1\frac{1}{4}$ (b) $1\frac{1}{4}$ (c) $-1\frac{3}{4}$ (d) $1\frac{3}{4}$

$\sqrt{10^2 - 6^2} =$

(a) ± 8 (b) ± 4 (c) 8 (d) 4

$\sqrt{18 \times 10 \times 10 \times 18} =$

(a) 100 (b) 10 (c) 180 (d) 18

$\sqrt{\sqrt{81}} =$

(a) 3 (b) 9 (c) 27 (d) 81

If $\frac{x}{2} = \frac{8}{x}$, then $x =$

(a) 16 (b) ± 4 (c) -4 (d) 4

If $x = \sqrt{\frac{1}{4}}$, then $x^3 =$

(a) $\frac{1}{64}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

أ/ فـريـد مـوسى
01032243340

Unit 1, Lesson 4 — The cube root

learn

- The product of a number by itself three times is the cube of that number.

  For example: 64 is the cube of 4 because $4 \times 4 \times 4 = 64$

- The symbol $\sqrt[3]{\ }$ (read as "the cube root of ") is used to designate the cube root.

  For example: $\sqrt[3]{64}$ designates the cube root of 64

- The cube root of a positive number is positive and the cube root of a negative number is negative.

  For example: $\sqrt[3]{64} = 4$ and $\sqrt[3]{-64} = -4$

  The cube root of any number has the same sign of this number.

- If a number is not a perfect cube, then you indicate its cube root by using the cube root symbol.

  For example: The cube root of 4 is $\sqrt[3]{4}$ because 4 is not a perfect cube

- $\sqrt[3]{a^3} = a$, **For example:** $\sqrt[3]{5^3} = 5, \sqrt[3]{(-5)^3} = -5$
- $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$, **For example:** $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$

[illegible]

- $\sqrt[3]{8} =$ ....
- $\sqrt[3]{216} =$ ....
- $\sqrt[3]{0.064} =$ ....
- $\sqrt{(-7)^2} - \sqrt[3]{(-7)^3} =$ ....
- $\sqrt[3]{27} =$ ....
- $\sqrt[3]{8} + \sqrt[3]{-8} =$ ....
- $\sqrt[3]{-27} =$ ....
- $\sqrt[3]{\frac{-8}{125}} =$ ....
- $\sqrt{4} - \sqrt[3]{-8} =$ ....
- $\sqrt[3]{512} =$ ....
- $\sqrt[3]{0.001} =$ ....
- $-\sqrt[3]{-1} - \sqrt{1} =$ ....

1. $\sqrt[3]{x} = 5$

2. $\sqrt[3]{x} = -\frac{1}{4}$

3. $\sqrt[3]{x} = -\sqrt{4}$

4. $\sqrt[3]{x} - 3 = -1$

1. $x^3 + 8 = 0$

2. $8x^3 - 7 = 20$

3. $x^3 - 10 = 990$

4. $(x + 3)^3 = 343$

5. $(5x - 2)^3 + 10 = 18$

$\sqrt[3]{0.001 \times \frac{1}{8}} =$

(a) $\frac{1}{2}$	(b) 2	(c) $\frac{1}{20}$	(d) 20

$\sqrt[3]{1000} \times \sqrt[3]{-0.008} =$

(a) $\frac{1}{2}$	(b) 10	(c) 2	(d) -2

$\sqrt[3]{-27} + \sqrt{12\frac{1}{4}} + \sqrt[3]{0.125} =$

(a) 1	(b) 0	(c) -1	(d) $\frac{11}{2}$

If $-\sqrt{25} = \sqrt[3]{y}$, then $y = \ldots$

(a) 5	(b) -5	(c) 125	(d) -125

If $x^3 = 64$, then $\sqrt{x} = \cdots$

(a) -4	(b) 4	(c) 2	(d) -2

If $x^3 = 27$, then $x^2 = \ldots$

(a) 3	(b) 6	(c) 9	(d) 81

$\sqrt[3]{x^6} = \sqrt{2}$

(a) x^3	(b) x^2	(c) x	(d) x^4

A cube with a volume of x^6 cm^3 . Find the sum of the lengths of its edges if x = 10

1 ....

....

....

....

....

Exercises (4)

[illegible]

$\sqrt[3]{-\frac{8}{27}} =$	$\frac{-\sqrt[3]{64}}{\sqrt{64}} =$ ....
$\sqrt{9} + \sqrt[3]{-8} =$	$\sqrt{27\sqrt[3]{27}} - \sqrt[3]{64} =$ ....
$\sqrt[3]{-343} =$	$\sqrt[3]{-27a^6} =$ ....
$\sqrt{27\sqrt[3]{27}} =$	$\sqrt[3]{\sqrt[3]{512}}$ ....

[illegible]

$\sqrt[3]{\dots\dots\dots\dots} = 4$	$\lvert\sqrt[3]{-125}\rvert = \sqrt{\dots\dots\dots\dots}$
$\sqrt{16} = \sqrt[3]{\dots\dots}$	$\sqrt[3]{64 + \dots\dots\dots\dots} = 5$

[illegible]

$x^3 + 27 = 0$

1 ....

....

....

$8x^3 + 7 = 8$

2 ....

....

....

$x^3 + 16 = \frac{3}{8}$

3 ....

....

....

....

$2x^3 - 5 = x^3 + 3$

4 ....

....

....

5. $(2x+1)^3 - 7 = 20$

....

....

....

EX 4 : Find the value of x in each of the following :

1. $x^3 = -8$

....

2. $x^3 = 64$

....

3. $x^3 + 5 = 32$

....

4. $\sqrt[3]{x} - 5 = 120$

....

[5] Choose the correct answer:

(1) $\sqrt[3]{(-8)^2} =$

(a) 2 (b) -2 (c) 4 (d) -4

(2) $\sqrt[3]{\left(\frac{1}{8}\right)^2} =$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(3) $\sqrt[3]{-64} + \sqrt{16} =$

(a) 8 (b) -8 (c) ± 8 (d) 0

(4) $\sqrt{25} - \sqrt[3]{-125} =$

(a) 10 (b) 0 (c) 5 (d) ± 5

(5) $\sqrt{(-2)^2} + \sqrt[3]{(-2)^3} =$

(a) -4 (b) 4 (c) 8 (d) 0

(6) $\sqrt[3]{3\frac{3}{8}} + \sqrt{0.25} =$

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) -2

Unit 2, Lesson 1 — The inequality

learn

- **The solution set of the inequality :** is the set whose elements satisfy the inequality and it is a subset of the substitution set.
- **Properties of inequalities**
- If $a < b$, then $a + c < b + c$
- If $a < b$, then $a - c < b - c$
- If $a < b, c$ is a positive number , then $ac < bc$
- If $a < b, c$ is a positive number , then $\frac{a}{c} < \frac{b}{c}$
- If $a < b, c$ is a negative number , then $ac > bc$
- If $a < b, c$ is a negative number , then $\frac{a}{c} > \frac{b}{c}$

$x + 2 < 5$

$\therefore x + 2 - 2 < 5 - 2$

$\therefore \quad x < 3$

When $x \in \mathbb{N}$

The S.S. $= \{2,1,0\}$.

When $x \in \mathbb{Z}$

The S.S. $= \{2,1,0,-1, \dots\}$

When $x \in \mathbb{Q}$

The S.S. $= \{x: x \in \mathbb{Q}, x < 3\}$

$2x - 5 > 5$

$\therefore 2x - 5 + 5 > 5 + 5$

$\therefore \quad 2x > 10$

$\therefore \quad \frac{2x}{2} > \frac{10}{2}$

$\therefore \quad x > 5$

When $x \in \mathbb{Z}$

The S.S. $= \{6, 7, 8, \dots\}$

When $x \in N$

The S.S. $= \{6, 7, 8, \dots\}$

When $x \in \mathbb{Q}$

The S.S. $= \{x: x \in \mathbb{Q}, x > 5\}$

$14 - 2x \leq 2$

....

$2x - 3 \geq 5$

....

$-11 \leq 3x - 5 < 4$

$\therefore 11 + 5 \leq 3x - 5 + 5 < 4 + 5$

$\therefore \quad -6 \leq \quad 3x \quad < 9$

$\therefore \quad \frac{-6}{3} \leq \quad \frac{3x}{3} \quad < \frac{9}{3}$

$\therefore \quad -2 \leq \quad x \quad < 3$

When $x \in \mathbb{Z}$

The S.S. $= \{-2, -1, 0, 1, 2\}$

When $x \in N$

The S.S. $= \{0, 1, 2\}$

When $x \in \mathbb{Q}$

The S.S. $= \{x: x \in \mathbb{Q}, -2 \leq x < 3\}$

$2(x + 2) < -2x + 4$

$5x - 2 < 1$

$5x - 10 < 2x - 1$

$3 - 3x \leq 9$

$10x + 2 \geq -18$

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01032243340

Exercises (1)

[illegible]

$3x - 2 < 1$

....

$4x + 2 \geq -10$

....

$2x + 1 \leq 9$

....

$-4x \geq -8$

....

$3 - 2x \geq 1$

....

$2 - 3x \leq 4$

....

$3x - 1 \geq 2x + 3$

....

$3x - 2 > x + 4$

....

$3(x + 2) < -x + 4$

....

$2(x + 1) < x + 4$

....

If $-x < 5$, then ....

(a) $x < -5$ (b) $x < 5$ (c) $x > -5$ (d) $x > 5$

If $x \in N$, then the S.S. of the inequality $-x > 3$ is .....

(a) $\emptyset$ (b) $\{-3\}$ (c) $\{-4, -5, \ldots\}$ (d) $\{4, 5, \ldots\}$

$\frac{x}{3} < 4$ is equivalent to ....

(a) $x < 12$ (b) $x > 12$ (c) $x < \frac{4}{3}$ (d) $x > \frac{4}{3}$

If $x \in Z$, then the S.S. of the inequality $20 < 5x < 25$ is ....

(a) $\emptyset$ (b) $\{4,5\}$ (c) $\{5\}$ (d) $\{4\}$

The S.S. of the inequality $-2x <$ zero in Q is ....

(a) $\emptyset$ (b) Q_+ (c) Q_- (d) Z_+

If $x > y$, then $\frac{1}{x} \ldots\ldots \frac{1}{y}$ where $x \neq 0$ and $y \neq 0$

(a) $\geq$ (b) $=$ (c) $>$ (d) $<$

If $x > 5$, then $-x$ ....

(a) > -5 (b) < -5 (c) ≥ -5 (d) < -9

1. If a teacher wants to buy 5 pens of the same type to distribute to the outstanding students in the class, so that he does not exceed spending 150 EGP, including 20 EGP for shipping expenses, write an inequality that represents the price of one pen and solve the inequality to find the maximum price for one pen

....

....

....

2. Find the smallest three consecutive even numbers whose sum is greater than 96 .

....

....

Unit 2, Lesson 2 | Multiplying the algebraic terms

learn

- When multiplying the algebraic terms, follow the following:
  1 - Multiply the coefficients using the signs rule.
  2 - Multiply the symbols by adding the indices of symbols which have like bases.
- For example: $(5X^2) \times (3x) = (5 \times 3) \times (X^2 \times X) = 15X^3$
- When dividing an algebraic term by another algebraic term, follow the following :
  1 - Divide the coefficients using the signs rule.
  2 - Divide the symbols taking care that the indices of like bases should be subtracted. (subtracting the indices of the divisor from the indices of the dividend)
  For example: $12a^3 \div 3a = 4a^{3-1} = 4a^2$

1	$2a^3 b \times 3ab =$	2	$\frac{3}{4}a^2 \times \frac{4}{3}a =$ ....
3	$\frac{2}{5}x^2 \times (-15x^3) =$	4	$\frac{2}{3}m^2n \times \frac{3}{4}n =$ ....
5	$2a \times (-3ab) =$	6	$-4l\,m^2 \times \frac{1}{2}l^2\,m^2 =$ ....
7	$-2x^2y \times 3xy^2 = =$	8	$21x \div (-3) = =$ ....
9	$-15x^2y^3 \div 5xy^2 = =$	10	$-24a^5b^3c^2 \div (-8a^2b) =$ ....

1	$5a^6 \div 5a = =$	2	$-8x^5y^3 \div (-y^3x^4) =$ ....
3	$\frac{-12x^3y^2}{4x^2y^2} = =$	4	$\frac{6a^2b^3}{-3ab^2} = =$ ....
5	$48a^4b^7 = 12a^2b^2 \times$	6	$10x^5 \div$ ... ... $= 2x^3$
7	$-2x^2y \times 3xy^2 = -$	8	$21x \div (-3) = =$ ....
9	$-15x^2y^3 \div 5xy^2 = =$	10	$-24a^5b^3c^2 \div (-8a^2b) =$ ....

Exercises (2)

$5x \times 3y =$	$(-3a) \times 7c =$ ....
$(-8y^5) \times (-7y^4) =$	$2x \times (-3x) =$ ....
$2xy \times (-3x^2) =$	$5x^3y^4 \times 2xy^2 =$ ....
$5ab^2 \times (-2a^2b) =$	$ab \times (-3a) \times (-2b) =$ ....
$2x^3 \times (-3x^2) \times (-5x^4) =$	$(-2x) \times 4x =$ ....

$6a \div 2 =$	$10c \div 2c =$ ....
$12x \div (-x) =$	$(-14x^2) \div 7x =$ ....
$(-25a^6) \div (-5a^2) =$	$24c^5 \div (-24c^5) =$ ....
$9x^5y^4 \div 6x^3y =$	$(-32a^3b^6) \div (-4a^3b^2) =$ ....

$\frac{2}{3}t^4 \times \frac{3}{2}t^4 =$	$\frac{2}{7}a^2 \times 21a^5 =$ ....
$\frac{6x^4y^2}{7} \times \frac{28xy^3}{3} =$	$3x^3 \times \frac{1}{6}x^2 =$ ....
$(-25a^6) \div (-5a^2) =$	$24c^5 \div (-24c^5) =$ ....
$9x^5y^4 \div 6x^3y =$	$(-32a^3b^6) \div (-4a^3b^2) =$ ....

(1) $3a^4b \times 5a^2b^2 \times 2a^3 =$

$30a^9b^3$ | $150a^{10}b^3$ | $30a^{10}b^2$ | $60a^{11}b^3$

(2) $(-3x^2y)^2 \times 2xy =$

$9x^2y^2$ | $6x^3y^2$ | $18x^5y^3$ | $-18x^5y^3$

(3) $(-6x^3y^2) \div 3x^2y = \cdots$

$-2x^2y^2$ | $-2xy$ | $2xy$ | $-2x^2y$

(4) If $2b$ cm is the edge length of a cube, then its volume $= \cdots$ cm^3

$8b^3$ | $4b^3$ | $2b^3$ | $4b^2$

(5) If the area of a rectangle is $24x^3$ cm^2 and its length is $8x^2$ cm, then its width is....

$4x^5$ | $4x$ | $3x^2$ | $3x$

$9a^5 = 3a \times$ ......

$36a^5b^8 = 12a^3b^2 \times$ ....

$-4c^3d^3 = 2cd^2 \times$...........

$81l^4 \div$ ... ... $= 27l^3$

Unit 2, Lesson 2 | Multiplying a monomial by an algebraic expression

learn

- Example : Find the product of each of the following:
- $b(-2a + a^2b) = -2ab + a^2b^2$
- $-3ab(5a - 2b + 3) = -15a^2b + 6ab^2 - 9ab$
- $(a^2 - ab - 2b^2) \times 4ab = 4a^3b - 4a^2b^2 - 8ab^3$
- $4(3x^2 + 5x) - x(x^2 - 7x + 8)$

$= (4)(3x^2) + (4)(5x) + (-x)(x^2) - (-x)(7x) + (-x)(8)$

$= 12x^2 + 20x + (-x^3) - (-7x^2) + (-8x)$

$= 12x^2 + 20x - x^3 + 7x^2 - 8x$

$= -x^3 + 19x^2 + 12x$

- $3a(2a - 4b) =$ ……………………
- $-2x(3xy - 5x) =$ ……………………
- $8x(y + 1) - x(y + 2) =$ ……………… ……………………………………
- $x(2y + 5) + x \times 5y =$ ……………… ……………………………………
- $(x + 5) \times 3x =$ ……………………
- $4x(5x^6 + 3y^5) =$ ……………………

1. $2a(a + 4b) - 3b(a - 3b) - (2a^2 + 8b^2)$,
   then find the numerical value of the result when : $a = 1$ and $b = -2$
   ……………………………………………………………
   ……………………………………………………………
   ……………………………………………………………
   ……………………………………………………………
2. $2x(3x - 2) + 3x(x + 1)$ **, then find the numerical value of the result when : $x = 3$**
   ……………………………………………………………
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[illegible] Find in simplest form

1. A swimming pool has dimensions 3x , 6x from the inside. A swimming pool has dimensions 4x , 5x+8 from the outside. Find the area of the walkway in terms of x .

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2. A rectangular water tank with dimensions x ، x ، $2x+3$ is filled with water. If water leaks out of it until the height of the water in it becomes $x+3$, what is the value of x if the volume of water leaking out of the tank is 27 cubic meters?

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[illegible] area of the shaded part in each of the following :

1.

	$2a+5$	
$3a$	$2a$	$a+1$

....
....
....
....
....
....

$2x + 8$

$2x$

$8x$ 5

2

$3x + 9$ $2x$

$5x$

3

$3x^2 + 2x + 7$

$4x$ $3x$ $x + 7$

4

Exercises (2)

$a(a+1) =$ ....

$a(a-2) =$ ....

$3x(7y-4z) =$ ....

$-3(y+3) =$ ....

$-2c(7-3c) =$ ....

$2x(3x^2+4y^2) =$ ....

$-5x(2x+y-3z) =$ ....

....

$3xy(2x^2-5x^2y-4y^2) =$ ....

....

$lm^2(l^2-3ml-4m^2) =$ ....

....

$\frac{1}{3}x^2(6x^2-9xy-3y^2) =$ ....

....

$3a(a-b)+4a(2a+b)$

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$3a(4a-2)-4a(3a-2)$

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$2a(3a-1)+3a(a+2)$,

then find the numerical value of the result when : $a=1$

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[illegible] area of the shaded part in each of the following :

1

$4a + 7$, $a + 2$, $6a$, $3a$

2

3

$x + 7$, x, $6x$

4

$2x^2 + 5x + 9$, $3x$, $2x$, $2x + 7$

Unit 2, Lesson 3 | Multiplying two binomials

learn

- $(x+5)(2x-3) = x(2x-3) + 5(2x-3)$

$$= 2x^2 - 3x + 10x - 15$$

$$= 2x^2 + 7x - 15$$

- Notice that:
  The two terms 5 and $2x$ are called the means.
  The two terms x and -3 are called the extremes.

- The square of an expression consisting of the sum of two terms =
  The square of the first $\pm$ 2 × The first × The second + The square of the second.
  Example : $(x-y)^2 = (x-y)(x-y) = x^2 - 2xy + y^2$

- The product of the sum of two terms and the difference between them :

$$(a+b)(a-b) = a^2 - b^2$$

The product of the sum of two terms and their difference = at the end of the book using the Excel the square of the first - the square of the second

1	$(3x+4)(2x-5) =$	2	$(5a-2b)(7a-3b) =$ ....
3	$(4x-3y)(3y+x) =$	4	$(2x+8)(2x-1) =$ ....
5	$(7x-4)(5x-7) =$	6	$(5x+3)(2x-9) =$ ....

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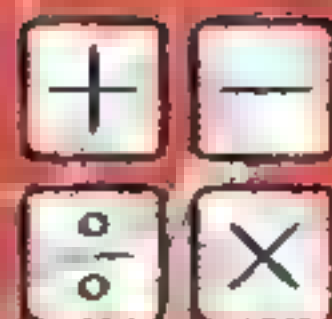
$(3a+5)^2 =$

$(2x-3y)^2 =$

$(3\text{ m}+2)^2 =$

$(5x-7y)^2 =$

$(2l-5)(2l+5) =$

$(5x+3y)(5x-3y) =$

$(a^2+2b)(a^2-2b) =$

$\left(\frac{1}{3}a-\frac{2}{5}b\right)\left(\frac{1}{3}a+\frac{2}{5}b\right) =$

$(2a+3b)(2a-3b) =$

$(3a-4b)(3a+4b) =$

1. $(x+4)^2-(x+2)(x+6) =$
2. $(x+5)(x-5)+(x-5)^2 =$
3. $(x-3)(x^2+4x-7) =$
4. $(-3x+x^2+3)(x-2) =$

[illegible]

1	$3502 \times 498 = (500+2)(500-2) = (500)^2 - (2)^2 = 250000 - 4 = 249996$
2	$(52)^2 =$ ....
3	$(195)^2 =$ ....
4	$3502 \times 498 = (500+2)(500-2) = (500)^2 - (2)^2 = 250000 - 4 = 249996$

[illegible]

1. $(2a+1)(5a+3) = 10a^2 +$ ....+ ....
2. $(3x+4)(2x-1) =$ ....+ .... $-\ 4$

[illegible] shaded part in each of the following:

1 ....

Figure 1: outer rectangle $x+7$ by $x+8$; inner unshaded square $x+2$ by $x+2$.

....

2 ....

Figure 2: rectangle of height $6x+2$, divided into parts of width $x+7$ and $x+1$.

3 ....

Figure 3: rectangle $5x+9$ by $3x+2$; cut-out square $x+3$ by $x+3$.

....

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$(3x+y)^2 =$

....

....

$(x-3y)^2 =$

....

....

$(2x+3y)^2 =$

....

....

$(-l-m)^2 =$

....

....

$(x+3)(x-3) =$

....

....

$(x-4)(x+4) =$

....

....

$(x-2)(x+2) =$

....

....

$(4m-7)(4m+7) =$

....

....

$(6x+2y)(6x-2y) =$

....

....

$(a^2+a)(a^2-a) =$

....

....

$(3x^2+5y^2)(3x^2-5y^2) =$

....

....

$\left(\frac{1}{2}x+\frac{1}{3}y\right)\left(\frac{1}{2}x-\frac{1}{3}y\right) =$

....

....

The middle term in the expansion of $(3x-1)^2$ is ...

$30a^9b^3$ (b) $6x$ (c) $-6x$ (d) $3x$

The middle term in the expansion of $(2a+3b)^2$ is

$-6ab$ (b) $6ab$ (c) $-12ab$ (d) $12ab$

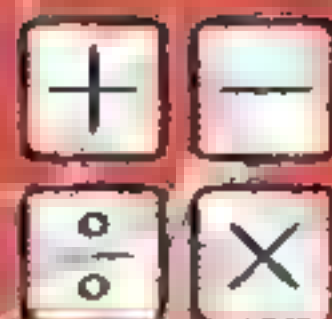
If $(2x+y)^2 = 4x^2+kxy+y^2$, then $k = \cdots$

6 (b) 8 (c) 4 (d) 2

If $x=-1$, then the numerical value of $(x+1)^2$ is ....

3 (b) 2 (c) 1 (d) zero

If $x^2 = 16, y^2 = 9$ and $xy = 12$, then $(x-y)^2 =$

1 (b) -1 (c) 165 (d) 49

If $(x+y)^2 = 26$ and $x^2 + y^2 = 20$, then $xy = \cdots$

12 9 6 3

If $x + y = 7$, then the numerical value of $x^2 + 2xy + y^2 = \cdots$

28 49 14 7

If $x - y = 3$ and $x + y = 5$, then $x^2 - y^2 = \cdots$

15 8 -2 2

If $x = \frac{4}{3}$, then $(x-2)(x+2) = \cdots$

$\left(\frac{4}{3}\right)^2 + 4$ $\left(\frac{4}{3}\right)^2 - 4$ $\left(\frac{4}{3}\right)^2 - 2$ $\frac{4}{3} - 2$

If $(x-3)(x+3) = x^2 + k$, then $k =$

-6 -9 6 9

If $(x-y)(2x+y) = 2x^2 + kxy - y^2$, then $|k| = \cdots$

4 3 1 -1

Example 5 :

Multiply, then find the numerical value of the expression when $x = 1$ and $y = -2$:

(1) $(x-5y)(x+5y) =$

....

....

....

(2) $(3x+y)(x+3y) =$

1

....

....

....

(3) $(x+4)(3x+2) =$

....

....

Reduce $(x-y)^2 + 2xy$, then find the numerical value of the result when $x = -1$ and $y = -2$:

2

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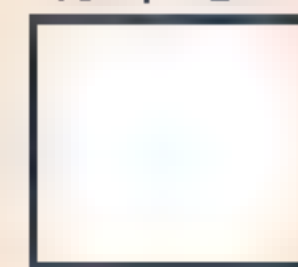
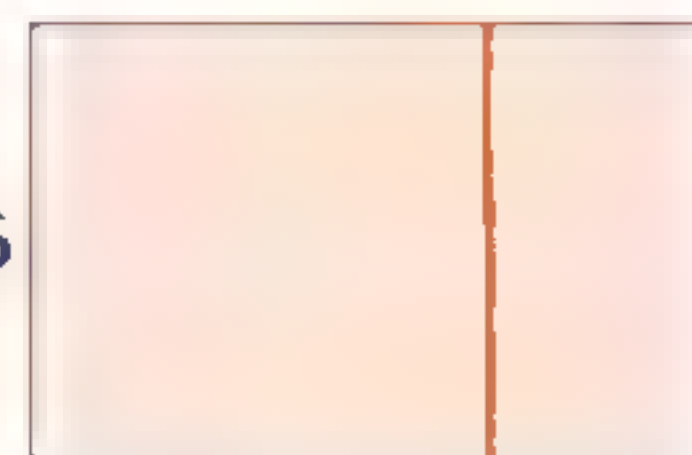
3	Reduce $= (2x-2)^2 + (x-2)(x+2)$, then find the numerical value of the result when $x = -1$: ....
4	Simplify to the simplest form $(2a-3)(2a+3)+7$, then find the numerical value of the result when $a = -1$: ....

1	....	$2x+5$ $x+4$ $3x+2$ $x+1$
2	....	$3x+5$ $x+3$ $2x+6$
3	....	$2x+8$ $x+3$ $x+1$ $x+1$

Unit 2, Lesson 4 | dividing an algebraic expression by an algebraic term

learn

- $\frac{6x^2+2xy}{2x} = \frac{6x^2}{2x} + \frac{2xy}{2x} = 3x + y$ where $x \neq 0$

[illegible]

$\frac{21x^2+14x}{7x} =$ ....

....

....

$(12x^4 + 8x^2) \div 4x =$

....

....

$\frac{10x^6y^4-8x^5y^3+2x^4y^2}{2x^4y^2} =$

....

....

$\frac{9x^4}{-3x^3} =$

....

....

$\frac{6x^3(3x^2-6x-9)}{9x^2} =$

....

....

....

$\frac{-8x^2(4x^2-2x-6)}{4x} =$

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$(16x^3y + 8x^2y^3 - 12x^2y)$ by $(-4x^2y)$

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$(14x^3 - 21x^2 + 7x) \div (-7x) =$

....

....

$\frac{3xy^2z-5x^2yz+2xyz^2}{xyz} =$

....

....

$\frac{49x^3 - 14x^2 + 21x}{-7x} =$

....

....

$\frac{18x^3+12x^2-6x}{-6x} =$

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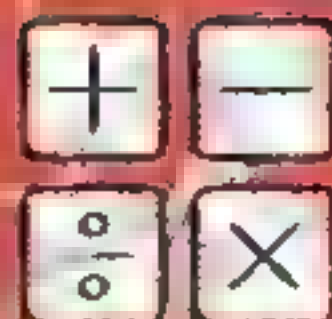
$\frac{3ab^2+9a^2b-6a^2b^2}{3abb} =$

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Exercises (5)

$4a^2 + 6a \div 2a =$

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$5a - 10 \div 5 =$

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$12a^2b + 20ab^2$ by $4ab =$

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....

$16a^3b^2 - 24a^2b^2$ by $4a^2b =$

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$12x + 15y$ by $-3 =$

....

....

$24x^3 - 18x^2 \div -6x^2 =$

....

....

$60x^6 - 48x^{10} - 12x^3$ by $-12x^3 =$

....

$32x^5 - 48x^3 + 72x^7 \div -8x^3 =$

....

$\dfrac{26x^2 + 14x^4}{2x} =$

....

....

$\dfrac{18m^4 + 32m^2}{-2m^2} =$

....

....

$\dfrac{48x^3 - 80x^2}{8x^2} =$

....

....

$\dfrac{9l^3m^4 - 18lm^2}{3lm^2} =$

....

....

$(x^2 + x) \div x =$ ....

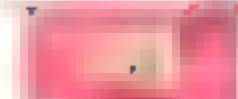
$x + 1$ | (b) $2x + 1$ | (c) x | (d) zero

$(15a + 5) \div 5 = \cdots$

$4a$ | (b) $3a + 1$ | (c) $10a$ | (d) $3a$

$(4a^3 - 2a) \div (-2a) = \cdots$

-1 | (b) $2a^2 + 1$ | (c) $-2a^2 + 1$ | (d) $-2a^2$

$(15x^4 + 5x^3) \div 5x^3 =$ ....

$4x^4$ | $3x + 1$ | $5x^2 + 1$ | $3x^2 + x$

$(3x^2y - \;\ldots\;) \div 3xy = x - 2y$

$-6xy^2$ | $6y^2$ | $6xy^2$ | $6x$

If $(6x^2y^3 + kxy) \div 6x = xy^3 - 12y, x \neq 0$, then $|k| = \cdots$

2 | (b) -2 | (c) 72 | (d) -72

1. If the area of a rectangle is $4x^4 + 8x^3 + 12x^2$ square units, and one of its sides is $4x^4$ units in length, find the other dimension in terms of x.

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2. A bakery prepares $(10x^3 + 15x^2 + 5x)$ pieces of biscuits and places them in boxes, each box containing $5x$ pieces. Find the number of boxes the bakery needs to package the biscuits in terms of x.

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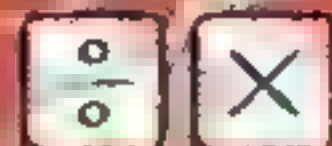
dividing an algebraic expression by an algebraic expression

learn

$$\begin{array}{r|l} x+2 & x^3 + \quad\quad x + 10 \\ x^2 - 2x + 5 & \ominus x^3 \ominus 2x^2 \\ \hline & -2x^2 + x + 10 \\ & \oplus{-2x^2} \oplus{-4x} \\ \hline & 5x + 10 \\ & \ominus 5x \ominus 10 \\ \hline & 0 \quad 0 \end{array}$$

Notice that:

There is no term with x^2 in the dividend, so we leave its place empty.

i.e. The quotient $= x^2 - 2x + 5$

$14x^2 + 25x + 6$ by $2x + 3$

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$2x^3 + x^2 - 19x + 10$ by $2x - 5$

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learn

If the expression $(2x^3 + 11x^2 + 12x + m)$ is divisible by $(x + 3)$,find the value of m

Divisor: $x + 3$ | Dividend: $2x^3 + 11x^2 + 12x + m$

Quotient: $2x^2 + 5x - 3$

$\ominus \quad \ominus$

$2x^3 + 6x^2$

$5x^2 + 12x + m$

$\ominus \quad \ominus$

$5x^2 + 15x$

$-3x + m$

$\oplus \quad \oplus$

$-3x - 9$

$m + 9$

$m + 9 = 0$

So, $m = -9$

Example

If the expression $(4x^2 + 11x + m)$ is divisible by $(4x - 1)$,find the value of m

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1	$x^2 - 5x - 14$ by $x - 7$	2	$2x^2 + 13x + 15$ by $x + 5$

A rectangle whose area is $(8x^2 + 6xy - 9y^2)\text{cm}^2$, if its width is $(4x - 3y)$cm then find its length, and calculate its perimeter when $x = 3$ and $y = 1$

1

Exercises (6)

$y^2 - 9y + 20$ by $y - 4$

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$x^2 + 5x + 6$ by $x + 2$

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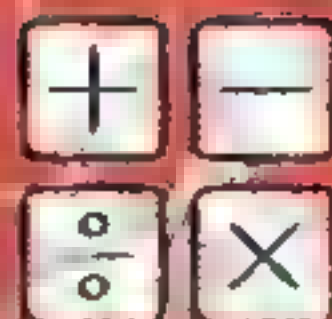
$x^2 - 5x - 14$ by $x - 7$

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$2x^2 + 13x + 15$ by $x + 5$

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$3x^2 + 2x - 8$ by $3x - 4$

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$x^2 - 6 - x$ by $x + 2$

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Example 2

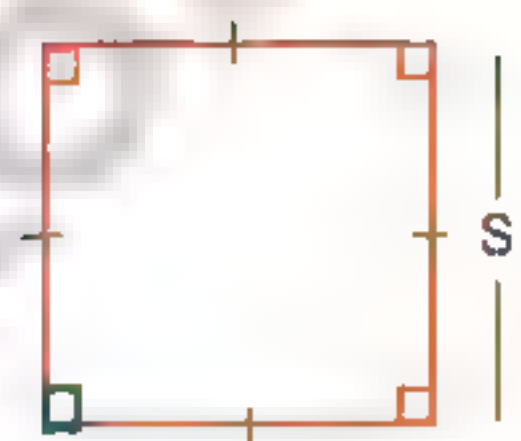
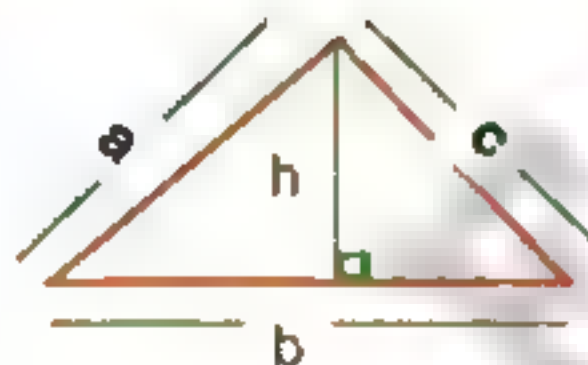
1. If the area of a rectangle is $(15x^2 + 11x - 14)\ cm^2$ and its width is $(3x - 2)$cm. Calculate its length.

2. If the area of a rectangle is $(2x^2 + 7x - 15)\ cm^2$ and its length is $(x + 5)$ cm. Find its width and calculate its perimeter when $x = 3$.

Unit 3, Lesson 1 Areas

learn

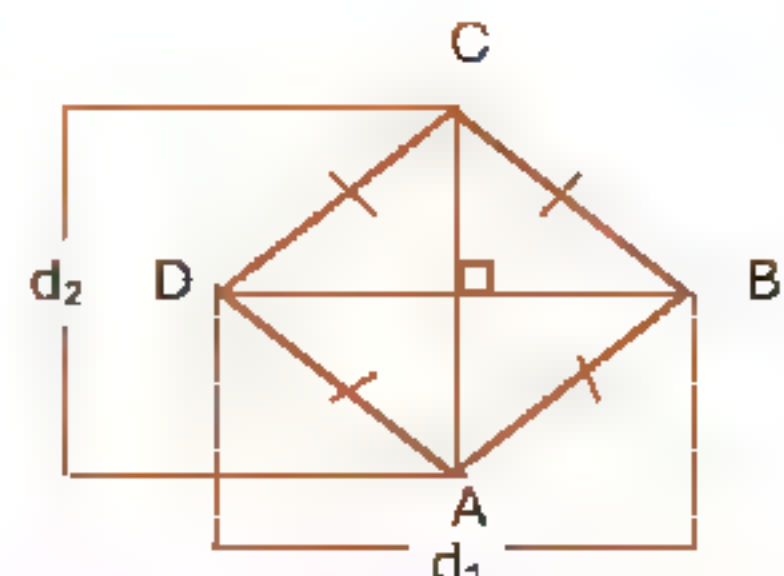
In previous years, you studied the mathematical formulas for finding the areas and perimeters of some geometric shapes, such as:

Parallelogram	Rectangle	Square
$P = 2(b_1 + b_2)$ $A = b_1 \times h$	$P = 2(\ell + w)$ $A = \ell x w$	$P = 4s$ $A = s^2$

Triangle	Rhombus
$P = a + b + c$ $A = \frac{1}{2} b \times h$	$P = 4s$ $A = s \times h$

The area of a rhombus given the lengths of its diagonals:

- The area of a rhombus is calculated as:
- $Area = \frac{1}{2} \times Diagonal_1 \times Diagonal_2$
- All sides of a rhombus are equal in length.
- Assuming the area is A , and the lengths

of the diagonals are d_1 and d_2 :

$A = \frac{1}{2} \times d_1 \times d_2$

Remember

- 1 cm = 10 mm , 1 dm = 10 cm , 1 m = 100 cm , 1 km = 1000 m
- 1 foot = 12 inches , 1 yard = 36 inches = 3 feet , 1 mile = 5280 feet

Example 1:

1 A rhombus has diagonals measuring 5 meters and 8 meters. Find its area.

The area of a rhombus =

$Area = \frac{1}{2} \times Diagonal_1 \times Diagonal_2$

$A = \frac{1}{2} d_1 \times d_2 = \frac{1}{2} \times 5 \times 8 = 20$

Therefore, the area of the rhombus is 20 m^2

2 A rhombus has a side length of 10 feet , a height of 9.6 feet , and one diagonal measuring 12 feet. Find the length of the other diagonal .

- The area of a rhombus = Side length × Height :

  $A = 10 \times 9.6 = 96\ ft^2$

$\therefore A = \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 12 \times d_2$

$\therefore 96 = 6 \times d_2$

$d_2 = \frac{96}{6} = 16$

Therefore , the length of the other diagonal is 16 ft .

3 A rhombus has diagonals measuring 8 inches and 10 inches. Find its area.

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4 In a rhombus ABCDABCDABCD, its diagonals intersect at M. MC=3x + 3 , and $MD = x$ find its area in terms of x .
Then, calculate the numerical value of the area when x = 7 .

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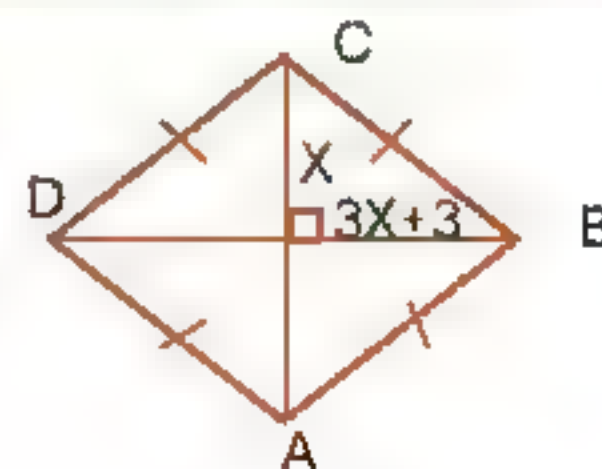
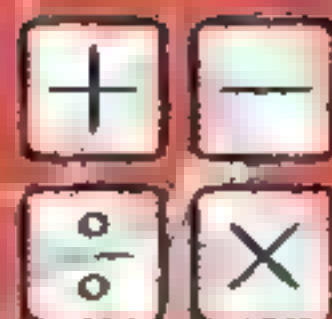

Area of a square given the length of its diameter

The Square is a Rhombus with Equal Diagonals, Hence:

- **Area of a Square:**

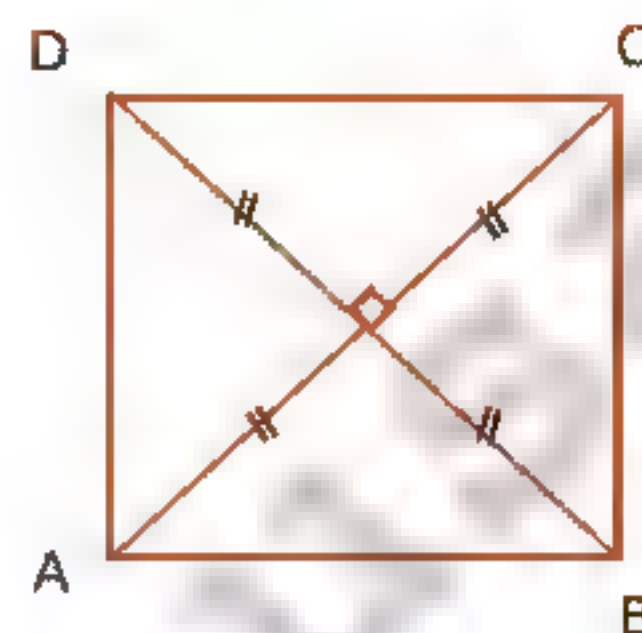
Area $= \frac{1}{2} \times$ Diagonal $\times$ Diagonal

Area $= \frac{1}{2} \times (\text{Diagonal})^2$

Let the area of the square be A, and its diagonal length d:

$A = \frac{1}{2} \times d^2$

Example: For a square with a diagonal of 4 cm:

$A = \frac{1}{2} \times 4^2 = \frac{1}{2} \times 16 = 8 \text{ cm}^2$.

Example 2 :

Which has a larger area?

1. A square with a diagonal of 12 cm or a rectangle with a length of 11 cm and a width of 7 cm?

- Area of the square (A_1):

$\therefore A_1 = \frac{1}{2} d^2 = \frac{1}{2} \times 12^2 = \frac{1}{2} \times 144 = 72$

Therefore, the area of the square is 72 cm^2

- Area of the rectangle (A_2):

$\therefore A_2 = \ell \times w = 11 \times 7 = 77$

Therefore, the area of the rectangle is 77 cm^2.

Conclusion:

Since 77 > 72, the rectangle has a larger area than the square.

2. A square with a diagonal of 10 feet and a parallelogram with a base of 6 feet and a corresponding height of 10 feet. Find the sum of their areas:

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The Area of a Trapezoid:

A trapezoid is a quadrilateral with only two parallel sides. Each of the parallel sides is called a "base," while the non-parallel sides are referred to as "legs."

For example, in the adjacent figure of trapezoid ABCD :

$\overline{AB}$ is the longer base (major base),

$\overline{DC}$ is the shorter base (minor base),

$\overline{AD}$ and $\overline{BC}$ are the legs.

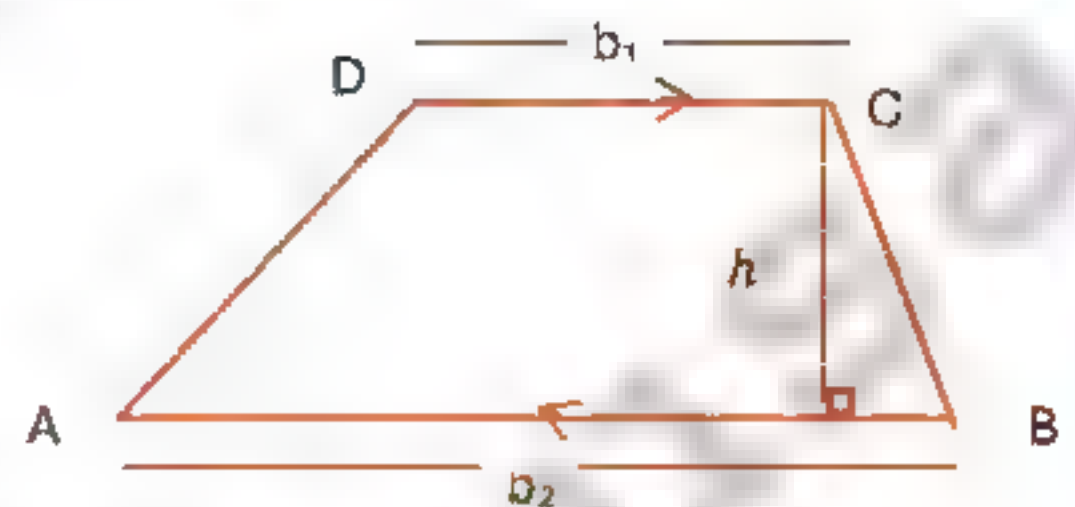
The formula to calculate the area of a trapezoid is:

Area $= \frac{1}{2} \times$(Sum of the lengths of the two parallel bases)$\times$(Height)

Let:

- A = area of the trapezoid,
- b1 = length of the smaller base,
- b2 = length of the larger base,
- h = height (the perpendicular distance between the two bases).

The formula can be expressed mathematically as:

$$A = \frac{1}{2}(b_1 + b_2) \times h$$

Note:

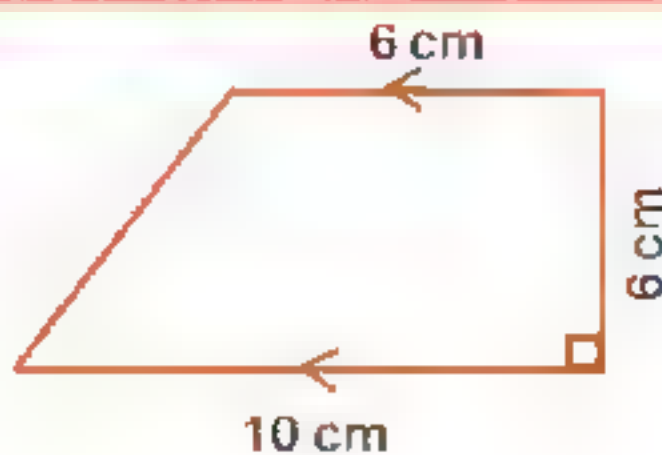
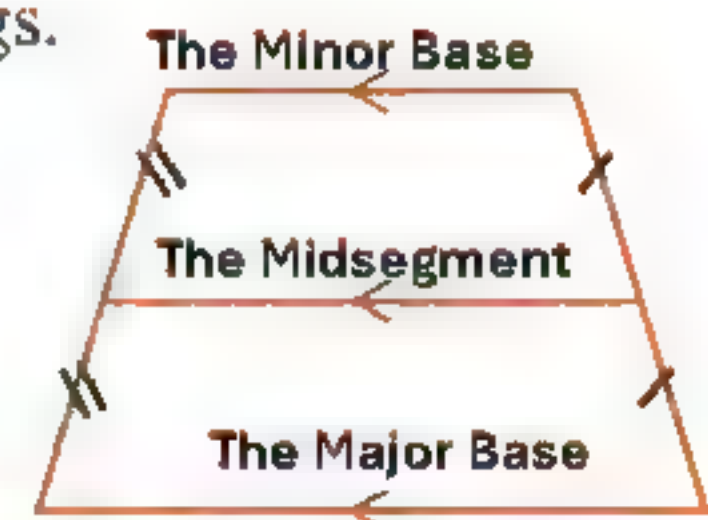
The Midsegment (Median) of a Trapezoid:

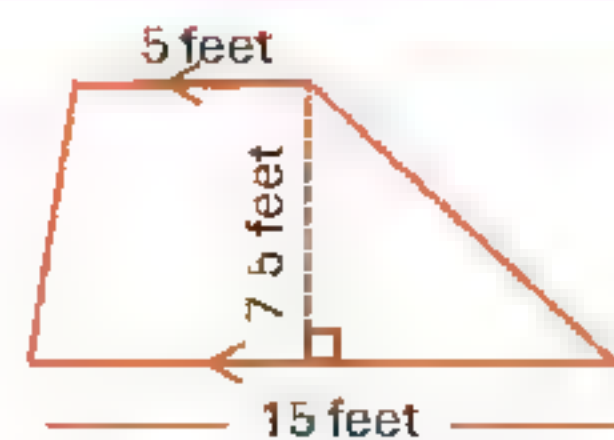
The midsegment is a straight line connecting the midpoints of its legs.

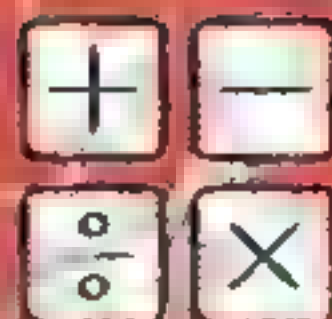
Length of the midsegment=

$= \frac{1}{2}$ *Sum of the lengths of the two parallel bases*

Area of the trapezoid = Length of the midsegment×Height

[illegible]

$\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore A = \frac{1}{2}(6 + 10) \times 6 = 48\ cm^2$

$\because \mathbf{A = \frac{1}{2}(b_1 + b_2) \times h}$

$\therefore \mathbf{A = \frac{1}{2}(5 + 15) \times 7.5 = 75}$ **square feet**

A trapezoid has a major base of 10 cm, a minor base of 6 cm, and a height of 8 cm. Calculate the area of the trapezoid.

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If the area of the trapezoid is 84 cm², the height is 7 cm, and the length of the minor base is 8 cm, calculate the length of the major base.

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A trapezoid has a major base equal to twice the length of its minor base, and the height is 10 cm. If its area is 200 cm², calculate the lengths of the major and minor bases.

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The major base of a trapezoid is 5 cm longer than the minor base, and the height is 12 cm. If its area is 180 cm², calculate the length of both bases.

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A trapezoid has an area of 54 square centimeters and a height of 9 cm. If the length of its minor base is 4 cm, find the length of its major base.

$\because \quad A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore \quad 54 = \frac{1}{2}(4 + b_2) \times 9$

$\therefore 4 + b_2 = 12$

$\therefore \quad b_2 = 8$

Thus, the length of the major base is 8 cm.

Exercises (1)

Question 1: Choose the correct answer from the options provided.

1. If the area of a rhombus is 100 square units, what is the product of the lengths of its diagonals?

(a) 25	(b) 50	(c) 100	(d) 200

2. If the area of a square is 450 square units, what is the length of its diagonal in units?

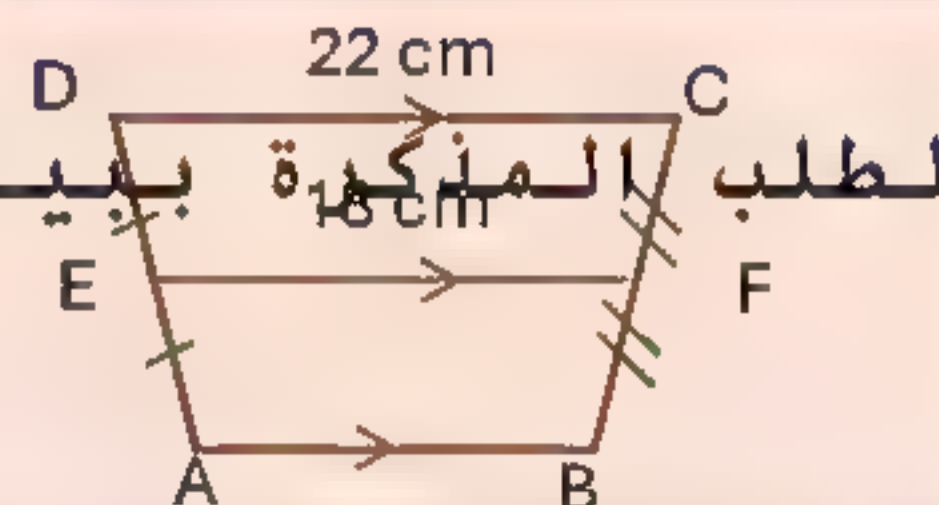
(a) 15	(b) 30	(c) 45	(d) 90

3. A trapezoid has the sum of its parallel bases equal to 16 cm, and its height is 5 cm. What is its area in square centimeters?

(a) 20	(b) 40	(c) 80	(d) 160

4. In the figure shown,

What is the length of AB in centimeters?

22 cm
D C
18 cm
E F
A B

(a) 14	(b) 20	(c) 26	(d) 28

5. A square has a side length of s and an area of A. What is the area of a square with a diagonal length of 2s?

(a) A	(b) 2A	(c) 4A	(d) A^2

1. A square has diagonals of 10 cm and 8 cm. Calculate its area.

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2. If the area of a square is 50 cm² and one of its diagonals is 10 cm, calculate the length of the other diagonal.

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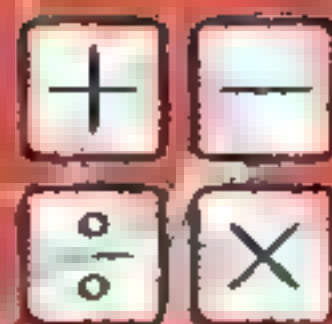
3. A square has an area of 72 cm². If one diagonal is 6 cm longer than the other, calculate the lengths of the diagonals.

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4. A square has one diagonal measuring 3x + 2 cm and the other 2x + 4 cm. If the area of the square is 56 cm², find the value of x.

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5. Find the length of the diagonal of a square whose area equals the area of a rhombus with diagonals of 4 meters and 25 meters.

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6. A rhombus has diagonals of 12 cm and 16 cm. Calculate its area.

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7. If the area of a rhombus is 72 cm² and one diagonal is 12 cm, calculate the length of the other diagonal.

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8. A rhombus has an area of 108 cm². If one diagonal is 6 cm longer than the other, calculate the lengths of the diagonals.

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9. A trapezoid has an area of 175 m², with the lengths of its parallel bases being 14 m and 21 m. Find its height.

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10. A trapezoid has an area of 225 in², one parallel base measuring 23 in, and a height of 7.5 in. Calculate the length of the other base.

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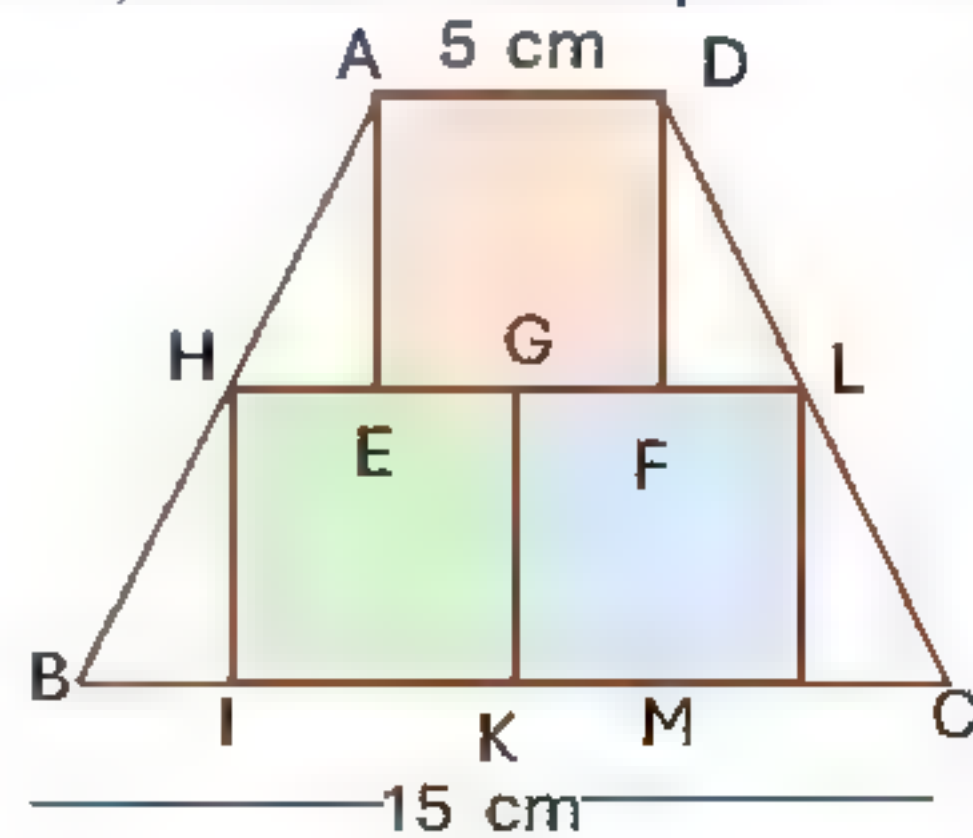
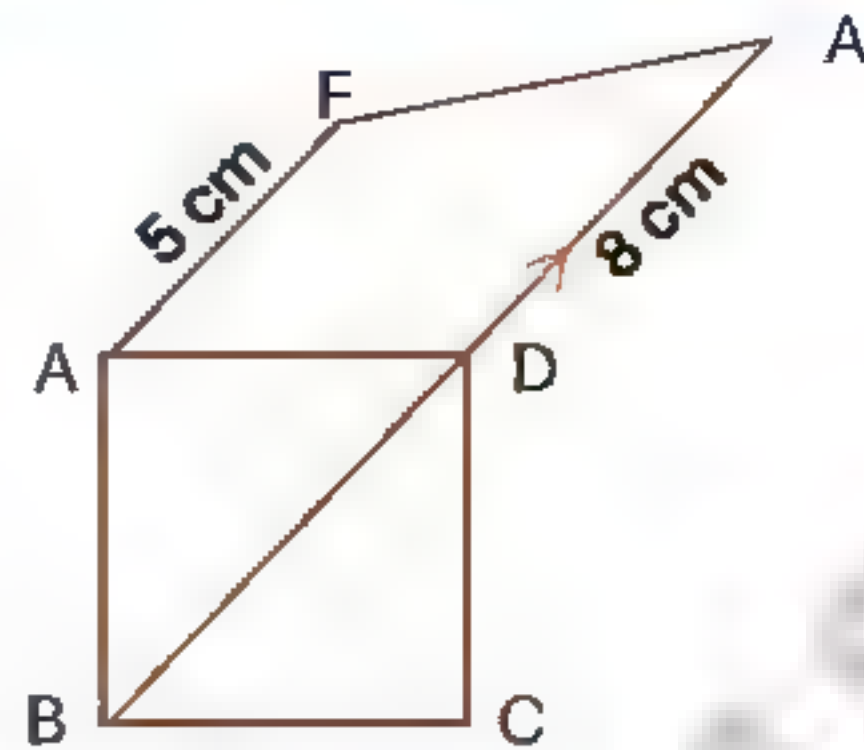
11. A trapezoid has an area of 315 cm², a height of 15 cm, and the ratio of its parallel bases is 3:4. Find the lengths of both bases.

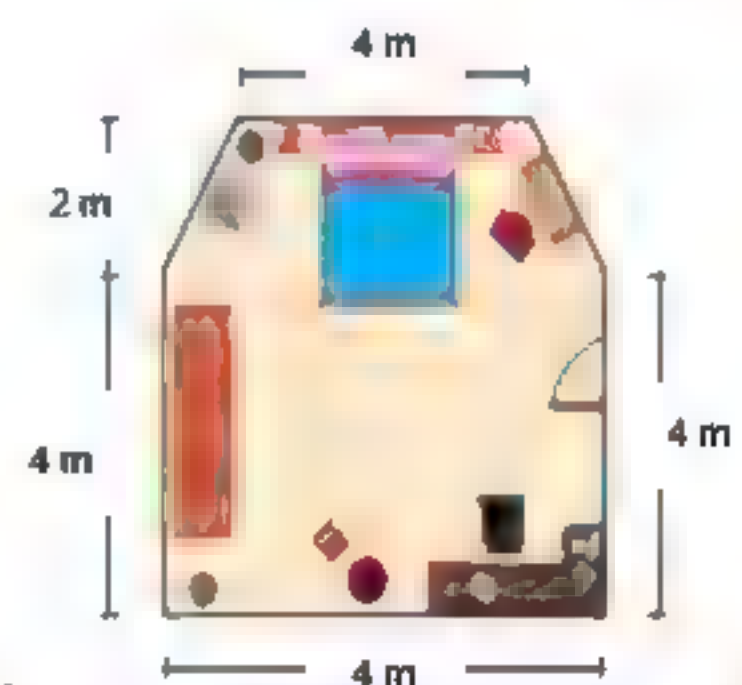
....

12. Find the area of trapezoid ABCD if GKML, HIKG, and AEFD are squares with equal side lengths.

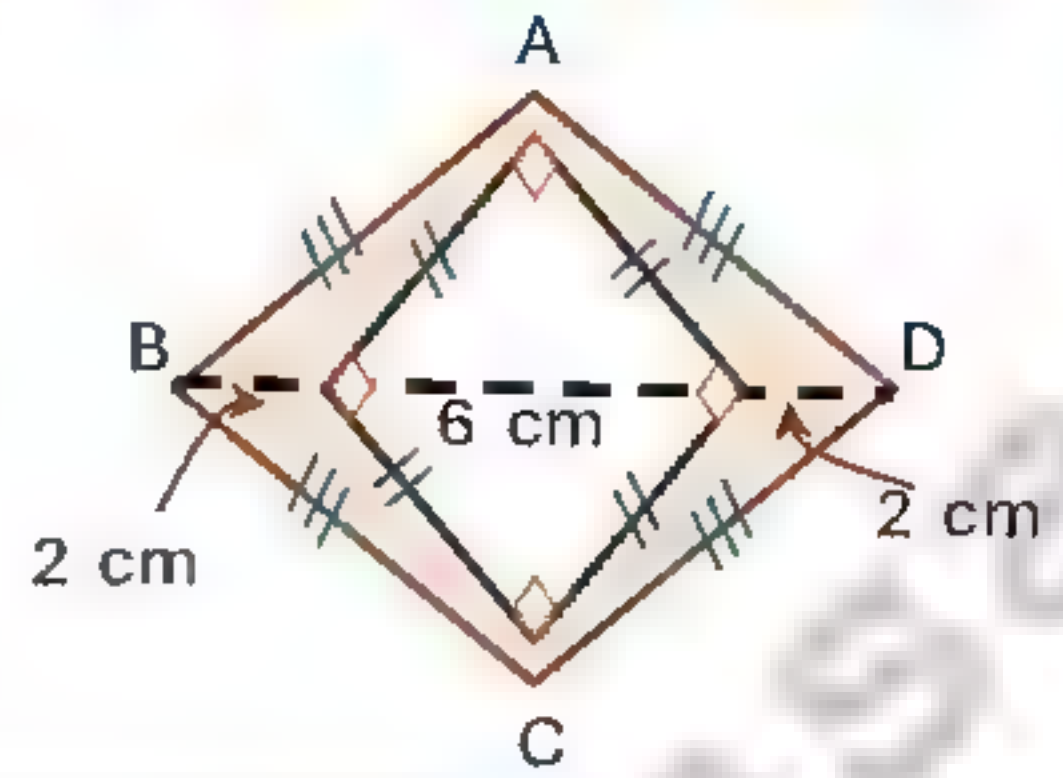
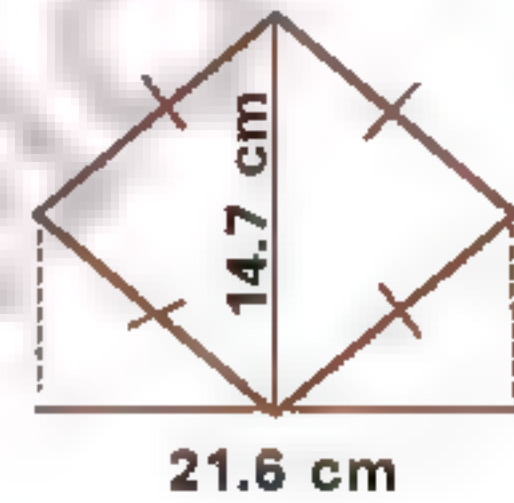
A 5 cm D
H G L
E F
B I K M C
15 cm

....

13. Calculate the area of trapezoid ABEF if ABCD is a square with a diagonal length of 6 cm.

14. A square plot of land has a diagonal of 8 km and an area equal to that of a rectangular farm with a width of 4 km. Find the length of the farm.

15. Two plots of land have equal areas; one is a rhombus with diagonals of 8 m and 27 m, and the other is a trapezoid with a height of 6 m. Find the length of the trapezoid's midsegment.

16. A room's dimensions are shown in the figure. The floor is to be tiled with ceramic tiles at a cost of 120 EGP per square meter. Calculate the cost of tiling the floor.

The arrangement of furniture according to the space of the house reflects aesthetic awareness and refined taste

18 Find the area of the shaded region in the given figure.

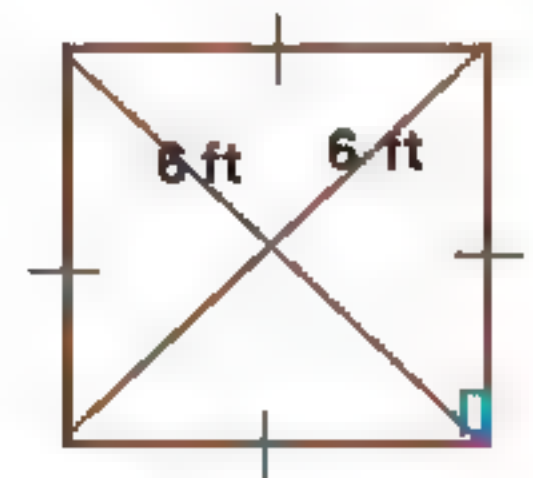
A, B, D, C, 6 cm, 2 cm, 2 cm

19 Calculate the area of the given shape.

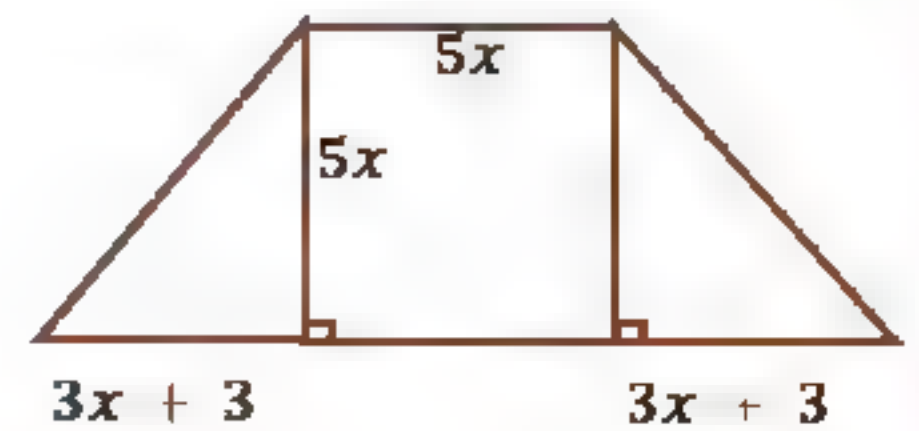
14.7 cm, 21.6 cm

20 Calculate the area of the given shape.

7.9 m, 6.5 m, 12 m

21 Calculate the area of the given shape.

6 ft, 6 ft

22 Express the area of the given shape in terms of x , then find its numerical value when x = 4 .

$5x$, $5x$, $3x + 3$, $3x + 3$

Unit 4
Lesson 2

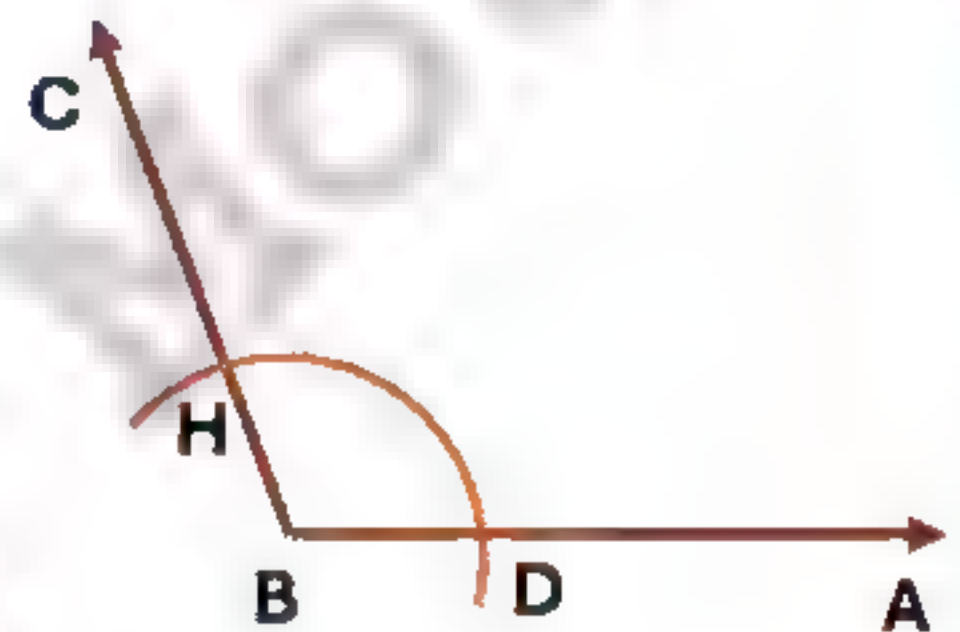
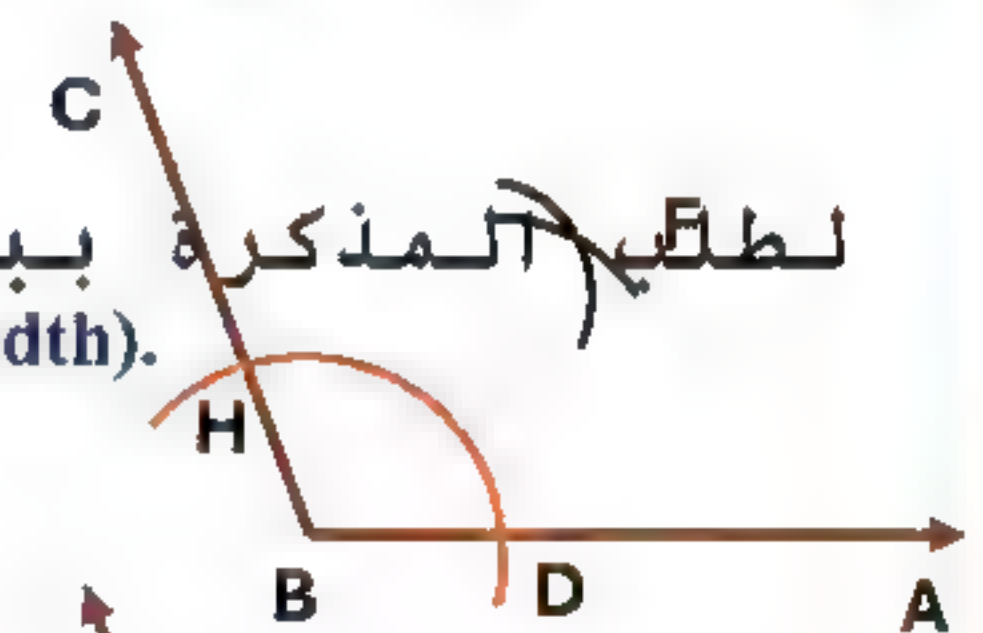
Geometric Constructions

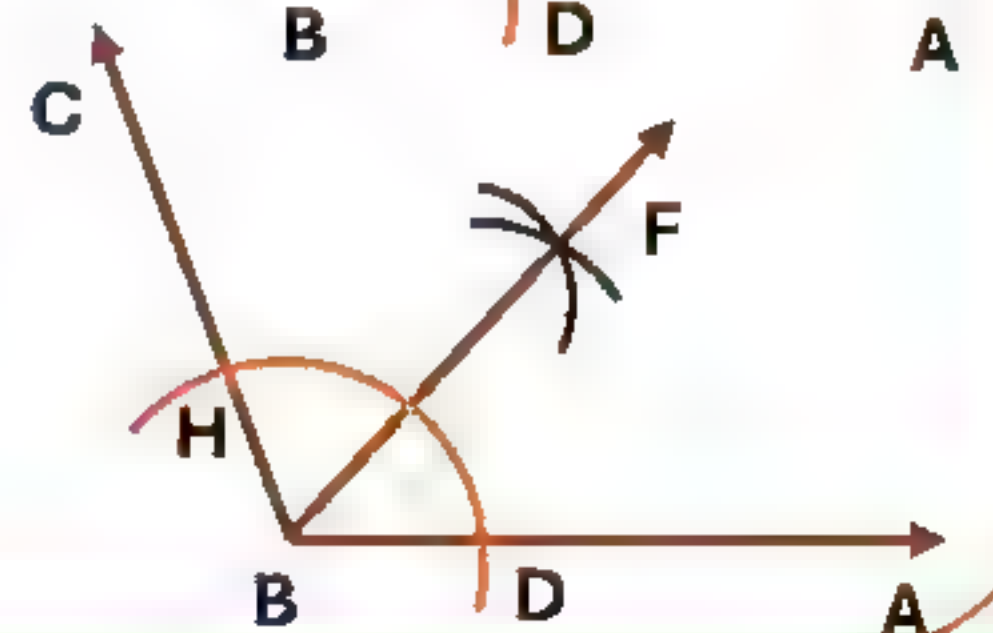
1- Construct the bisector of a given

Given: Angle $\angle ABC$ is known.

Required: Draw the bisector $M(\angle ABC)$ using a compass.

Steps:

1. Place the point of the compass at the vertex of the angle B and open it to a suitable width. Draw an arc that intersects $\overrightarrow{BA}$ at point D and $\overrightarrow{BC}$ at point H.
2. Place the point of the compass at both points D and H with the same compass width (or a suitable width). Draw two arcs that intersect at point F.

3. Draw the line $\overrightarrow{BF}$, which is the bisector $M(\angle ABC)$.

Example 1

Draw an angle of 70° and then bisect it.

1

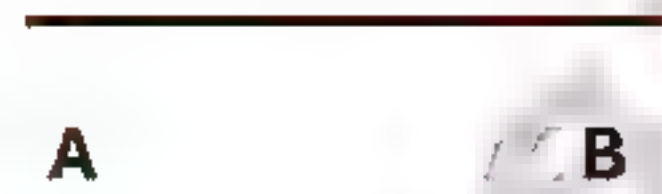
Bisecting a Line Segment or Drawing an Axis of Symmetry:

Given: The line segment $\overline{AB}$ is known.

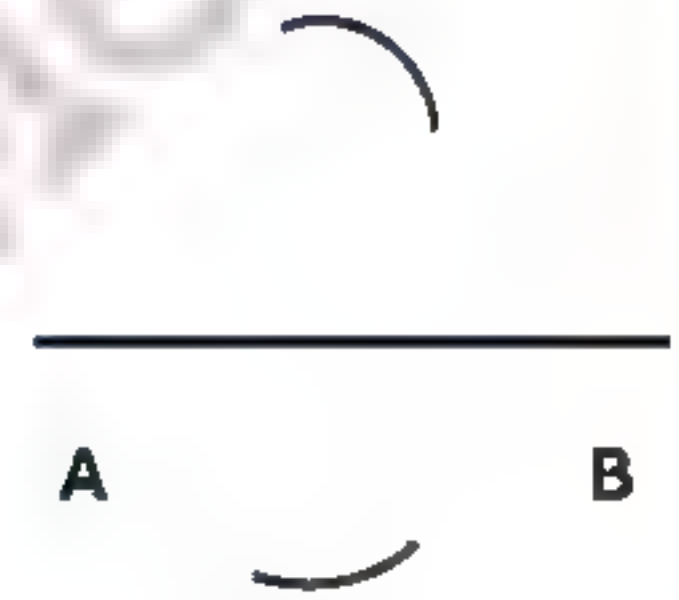
Required: To bisect $\overline{AB}$.

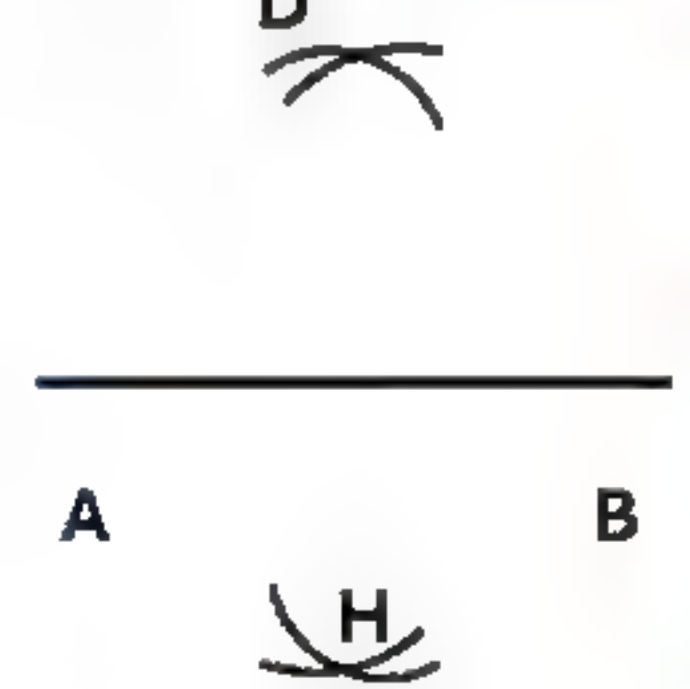
Steps to Bisect the Line Segment:

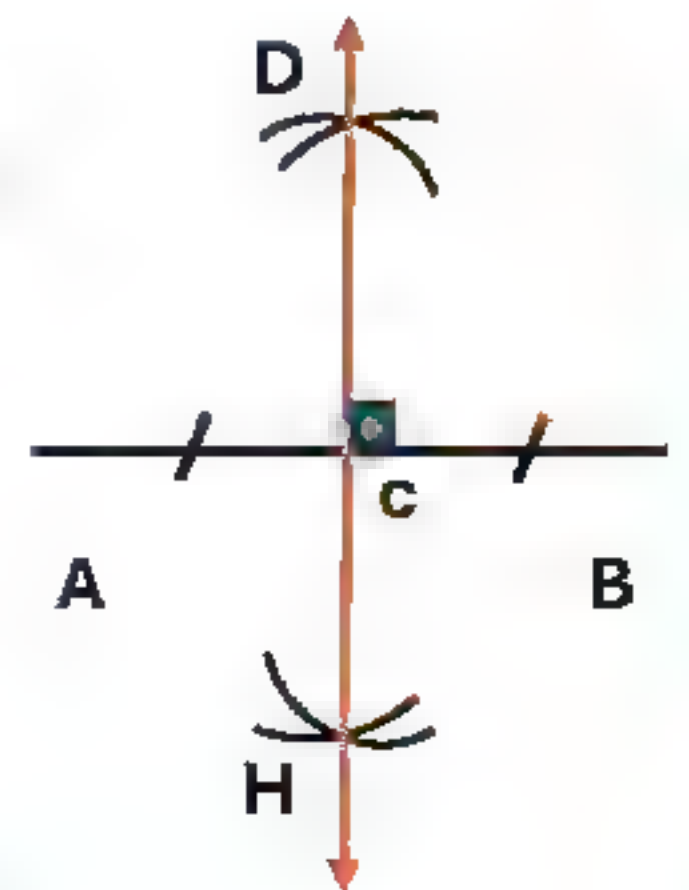
1. Draw the line segment $\overline{AB}$:

A B

Using a straightedge, draw the segment $\overline{AB}$ with its endpoints A and B.

2. Place the compass at point A:
   Set the compass to a radius slightly larger than half the length of $\overline{AB}$. Draw two arcs on opposite sides of the segment.

A B

3. Place the compass at point B:
   Without changing the compass radius, draw two arcs on opposite sides of $\overline{AB}$. These arcs will intersect the arcs from Step 2 at points D and H.

D A B H

4. Draw the perpendicular bisector $\overleftrightarrow{HC}$:
   Using a straightedge, connect the intersection points D and H. This line, $\overleftrightarrow{HC}$, intersects $\overline{AB}$ at point C.
   - Point C: This is the midpoint of $\overline{AB}$.

D A C B H

Note:

The perpendicular bisector $\overleftrightarrow{HC}$ is also the axis of symmetry for the line segment $\overline{AB}$. It is perpendicular to $\overline{AB}$ and divides it into two equal parts.

Example /

1

Draw the line of symmetry for a line segment AB with a length of 8 cm

2

Using geometric tools, draw $\angle ABC$ measuring 110°, then draw $\overrightarrow{BD}$ to bisect it into two equal angles.

3

Draw $\angle ABC$ measuring 100°, then bisect it using a compass and ruler

Exercises (2)

Example 1

1. Draw an angle measuring 70° using a protractor, then bisect it using a compass and ruler

2. Draw an angle measuring 100° using a protractor, then bisect it using a compass and ruler

3. Draw a right angle, then bisect it using a compass and ruler.

4. Draw an angle $\angle FCD$ measuring 120°, then draw $\overrightarrow{CH}$ to bisect $\angle FCD$

5 Draw $\angle ABC$ measuring 80°, then bisect it into two equal angles

(2) Draw the line of symmetry for a given line segment

1 Draw the line of symmetry for a line segment AB with a length of 5 cm

2 Draw a line segment with a length of 6 cm, then bisect it.

3 Draw a line segment with a length of 4 cm, then draw the line of symmetry for it.

Geometric Constructions

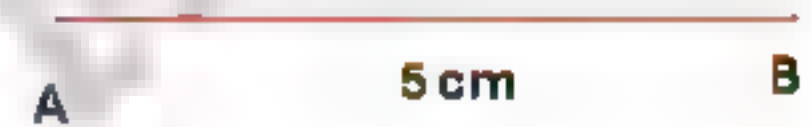
First: Draw a triangle with the known lengths of its sides

Drawing a Triangle When the Lengths of Its Sides Are Known:

To draw triangle $\triangle ABC$, where : $AB = 5$ cm , BC = 4 cm , AC = 3 cm

Follow these steps :

Using a ruler, draw a straight line $\overline{AB}$ with a length of 5 cm.

A 5 cm B

Set the compass for 4 cm

- Open the compass to a radius of 4 cm.
- Place the compass point at B, and draw an arc.

A 5 cm B

Set the compass for 3 cm:

- Open the compass to a length of 3 cm.
- Place the compass point at A, and draw an arc that intersects the first arc at point C.

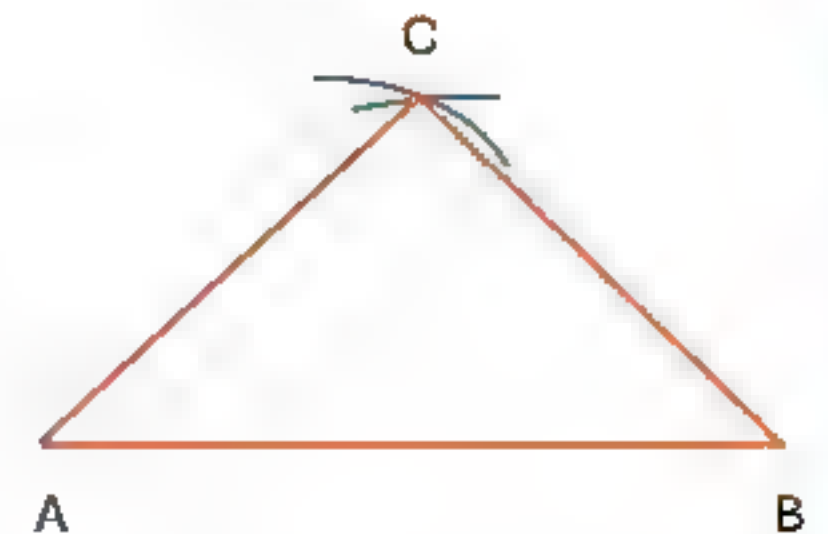
A 5 cm B

Complete the triangle:

- Connect A to C, and B to C, to form triangle ABC .

The triangle ABC now has side lengths

AB =5 cm , BC = 4 cm , AC = 3 cm .

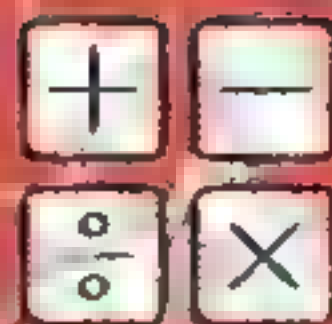
Example 1

Draw the triangle ABC where AB = 5 cm, BC = 3 cm, AC = 4 cm:

1

Draw the triangle ABC where AB = 4 cm, BC = 4 cm, AC = 4 cm:

2

Draw the triangle ABC where AB = 3 cm, BC = 3 cm, AC = 5 cm:

3

of the Included Angle

To draw triangle ABC where $AB = 4$ cm , $AC = 3$ cm , and $m(\angle BAC) = 65°$

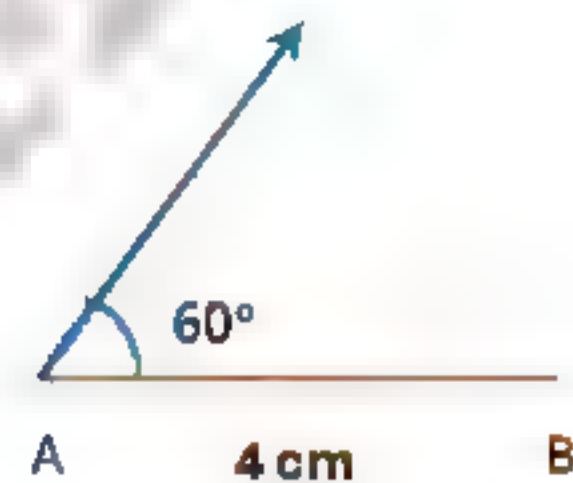
follow these steps:

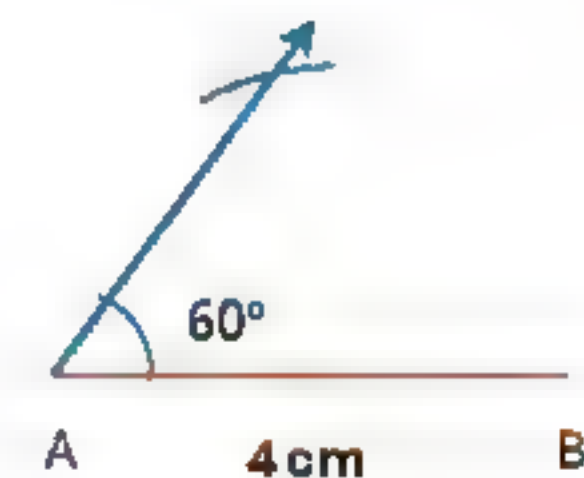
Draw the base AB :

Use a ruler to draw the line segment $\overline{AB}$ with a length of 4 cm.

A 4 cm B

Measure the angle ∠BAC :

- Place the protractor at point A.
- Mark an angle of 65° and draw a ray starting from A along the marked direction.

Mark the length of AC :

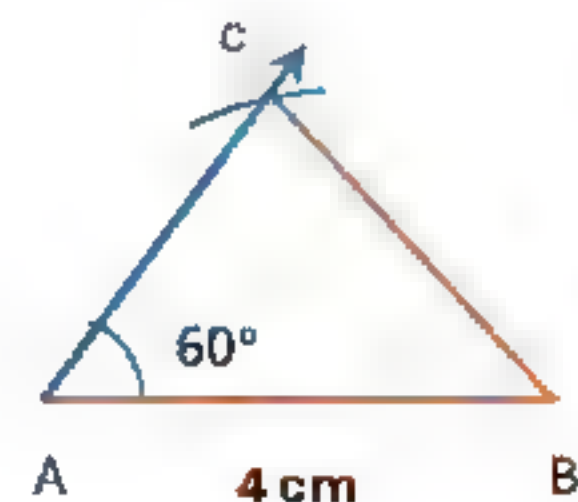
- Set the compass to a radius of 3 cm.
- Place the compass point at A, and draw an arc that intersects the ray at a point.
  This intersection point will be C .

Complete the triangle:

- Use the ruler to connect C to B.

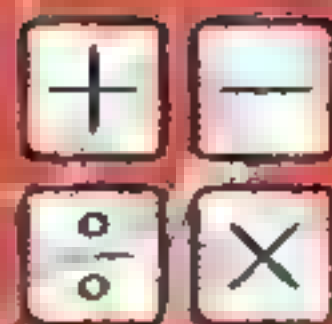
The resulting triangle ABC will have

AB = 4 cm , AC = 3 cm , and $m(\angle BAC) = 65°$

Example /

1

Draw triangle ABC where AB=10 cm ,BC = 8 cm, $m(\angle B) = 90°$

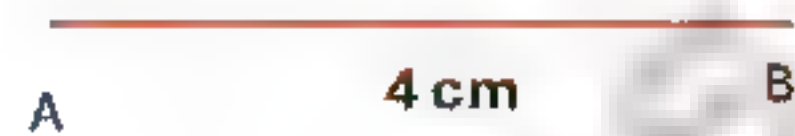
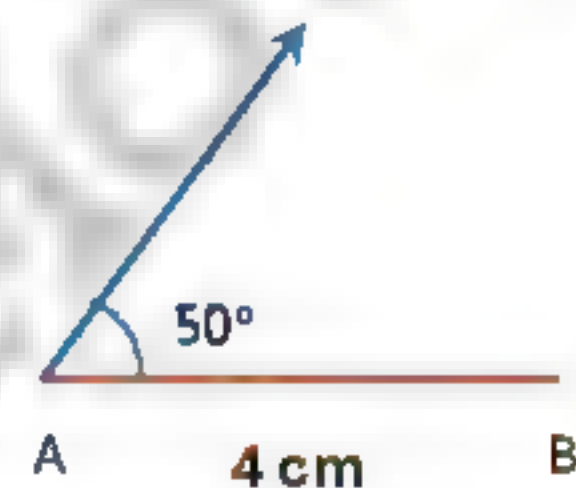
2

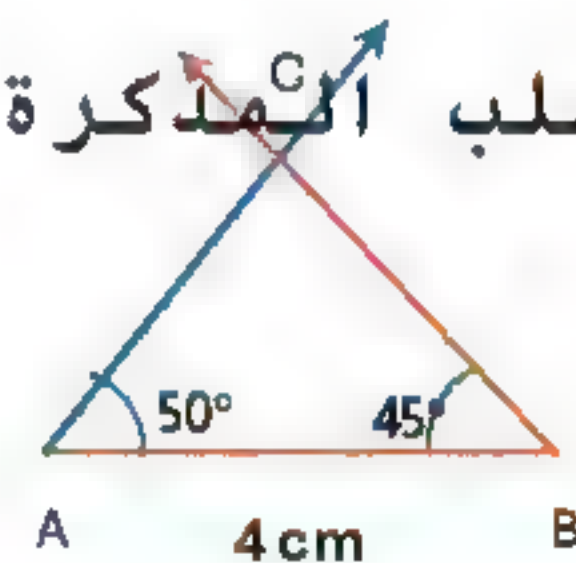
Draw triangle ABC where AB=5 cm, BC=6 cm, $m(\angle A) = 60°$

Drawing a triangle given the measures of two angles and the length of the side between their vertices.

To draw triangle ABC where AB = 4 cm , m (∠B) = 45°, m (∠A) = 50°,

follow these steps:

Use a ruler to draw a line segment $\overline{AB}$ with a length of 4 cm.

A 4 cm B

Using a protractor, from point A, mark an angle of 50° , and then draw a ray to define the angle

From point B, mark an angle of 45° using the protractor, and then draw a ray to define this angle, which intersects the first ray at point C.

Thus, triangle ABC is formed,
where m (∠B) = 45°, m (∠A) = 50°, $AB = 4\ cm$

Example 3

draw triangle ABC where AB = 5 cm , m (∠A) = 50°, m (∠B) = 40°

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draw triangle ABC where AC = 5 cm , m (∠A) = 40°, m (∠C) = 60°

2

Using geometric tools, draw triangle ABC where AB = 4 cm , BC = 3 cm , and m (∠B) = 90° . Then bisect $\overline{AC}$ at point D . Is $\frac{1}{2}$ AC = BD ?

3

Exercises (3)

(1)

1. Draw triangle $\triangle ABC$: AB =7cm , BC = 5 cm , $m(\angle ABC)=80°$,
Determine the type of triangle based on its angles.

2. Draw triangle $\triangle ABC$: $m(\angle ABC) = 42°$, $m(\angle ACB) = 38°$, BC = 6cm
Determine the type of triangle based on its side lengths.

3. Draw triangle $\triangle XYZ$: XY = 6 cm, YZ = 4 cm , XZ = 5 cm
Determine the type of triangle based on its angles.

4. Draw an equilateral triangle $\triangle ABC$: Each side AB = 5 cm.

5 Draw triangle $\triangle ABC$: $AB = 8$ cm , $m(\angle B) = 50°$, $m(\angle A) = 70°$

- Bisect $\overline{AC}$ at point D and $\overline{BC}$ at point E.
  Prove by measurement that AB = 2D.

6 Draw $\angle ABC$ with $m(\angle ABC) = 60°$:

- Bisect the angle using a ruler and compass to create $(\overrightarrow{BD})$.
- Bisect $\angle CBD$ and $\angle ABD$ using bisectors $\overrightarrow{BF}$، $\overrightarrow{BE}$, respectively.
  Prove by measurement that $m(\angle ABF) = 3m(\angle CBF)$.

7 Draw triangle $\triangle ABC$: $AB = 3$ cm , $BC = 5$ cm , $m(\angle B) = 40°$.

8

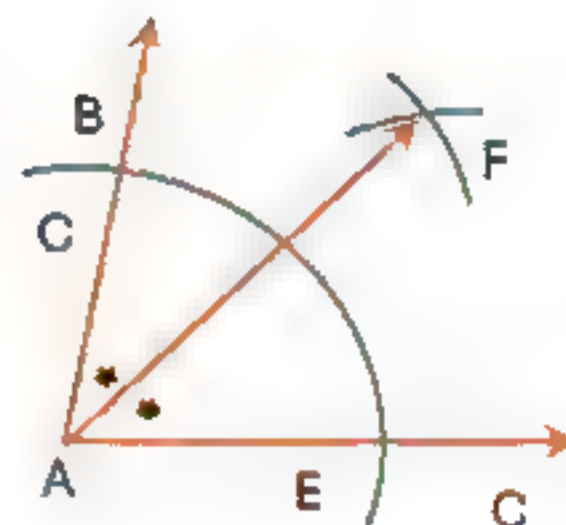
Draw triangle ΔABC: $AC = 5$ cm , $m(\angle C) = 30°$, $m(\angle A) = 70°$.

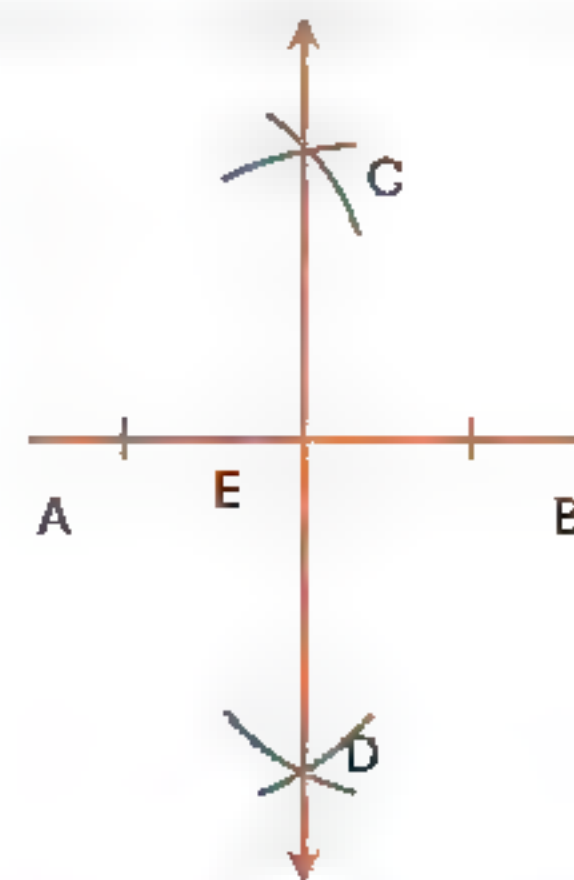
Question 2: Choose the correct answer from the options provided.

1 When bisecting $\angle BAC$ using a compass, find: $m(\angle BAF) = ...$

(a)	$m(\angle BFA)$	(b)	$m(\angle EAF)$
(c)	$m(\angle EFA)$	(d)	$m(\angle BAC)$

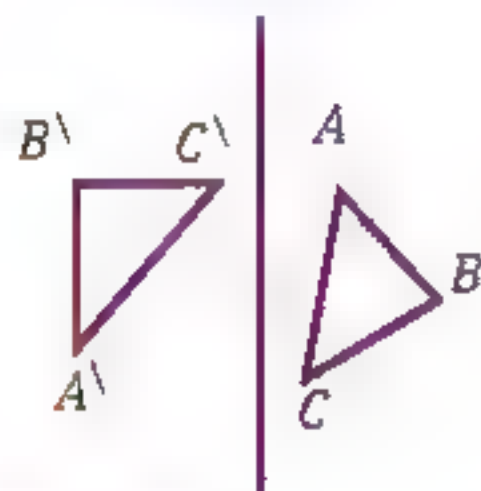
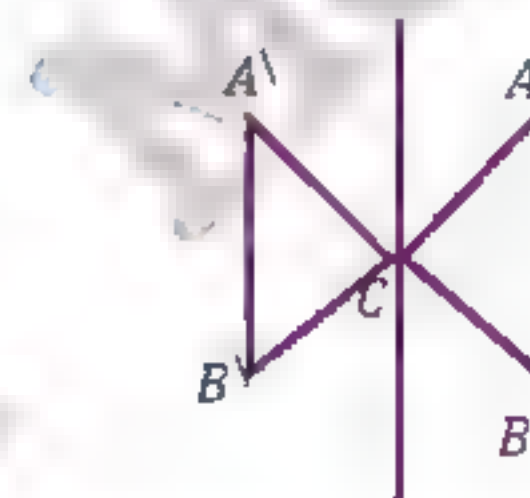
2 The length of $\overline{EF}$ must equal the length of:

(a)	$\overline{DF}$	(b)	$\overline{AD}$
(c)	$\overline{AE}$	(d)	$\overline{AF}$

3 When bisecting a line segment $\overline{AB}$ using a compass, the following must hold:.....

(a)	$AC < \frac{1}{2}AB$	(b)	$AC < AD$
(c)	$AC > \frac{1}{2}AB$	(d)	$AC < AE$

geometric transformation

geometric transformation

Definition: A geometric transformation is the process of moving a geometric shape from one position to another without changing its dimensions, meaning the shape and its image are exactly identical.

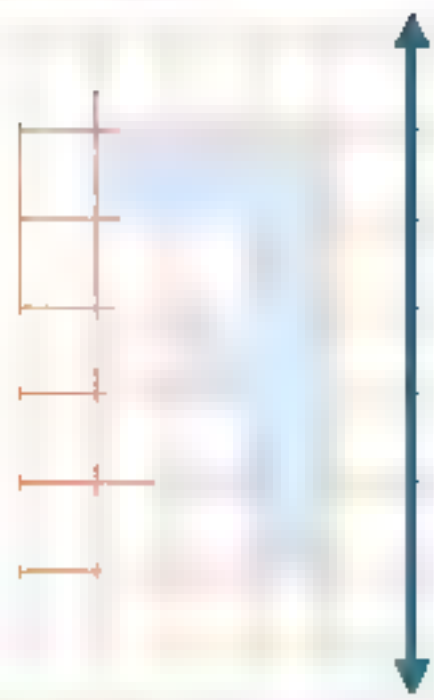
Types of Geometric Transformations:

Rotation - Translation – Reflection

- **Reflection:**
  It's as if the shape is in front of a mirror, where the shape ≡ its image.

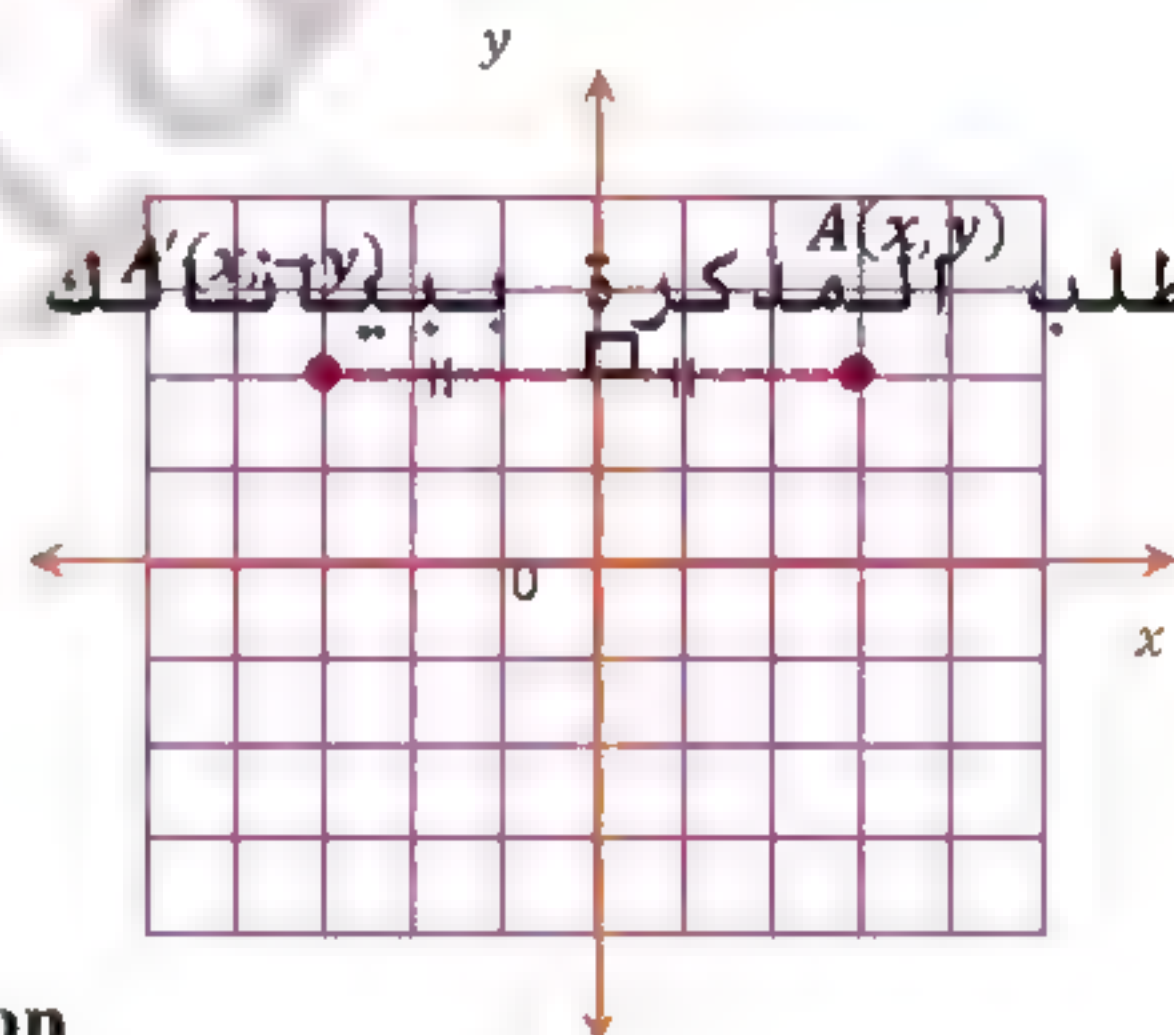
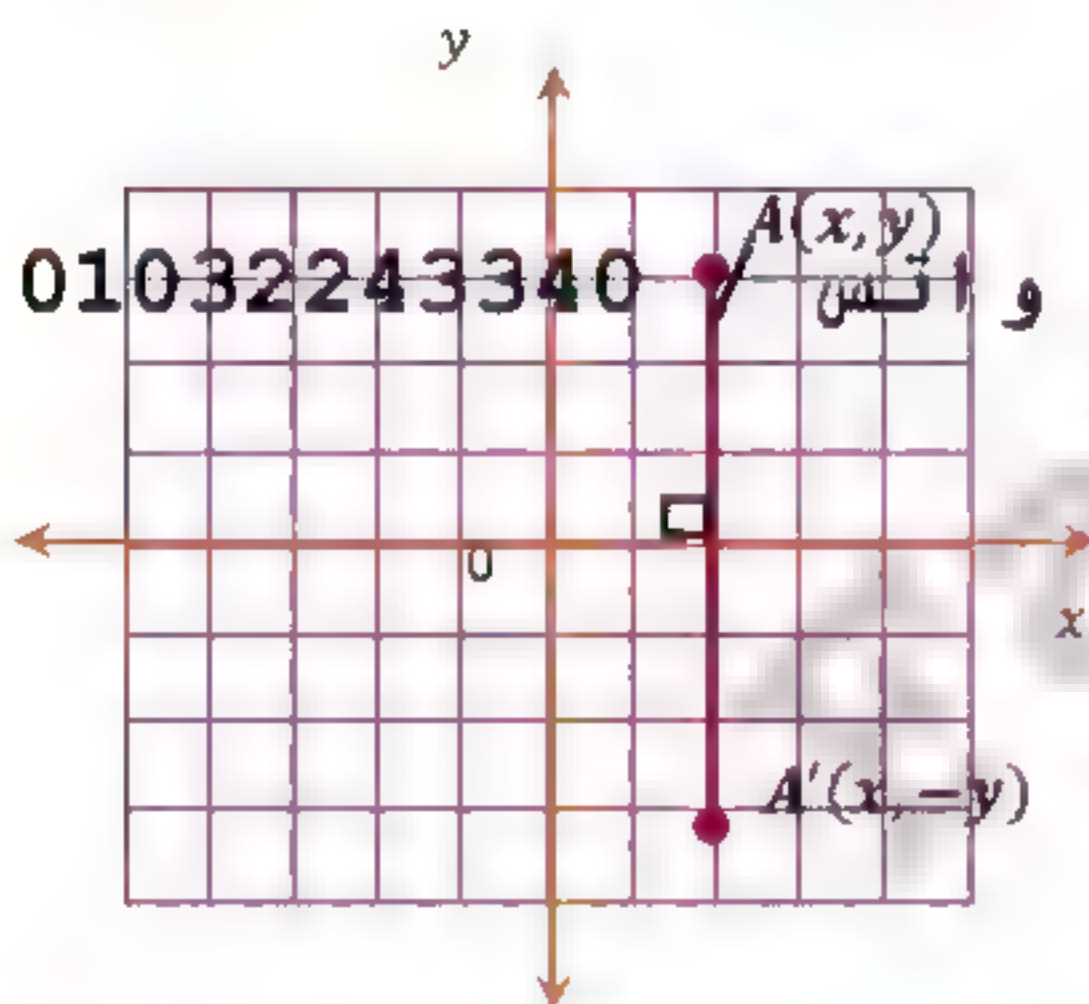
- **Translation:**
  It's as if the shape has moved
  (shifted from one place to another while maintaining the same shape).
- **Rotation:**
  The shape rotates around a point.
- Thus, a geometric transformation is the process of transforming each point A within the plane into its image A′ within the same plane.

- **Geometric transformations**

If all the points of a geometric figure move according to a specific system, we get an image of this figure in a new position. This figure is said to be under the influence of a geometric transformation. Examples of geometric transformations include reflection, translation, and rotation.

Reflection in a line

It is the formation of a mirrored image of a shape across a line called the line of reflection.

between them

- $A(x, y) \xrightarrow[x-axis]{\text{Reflection}} A'(x, -y)$, $A(x, y) \xrightarrow[y-axis]{\text{Reflection}} A'(-x, y)$

- The image of the point $(\chi, y) \xrightarrow[\text{at the origin}]{\text{Reflection}} (-\chi, -y)$

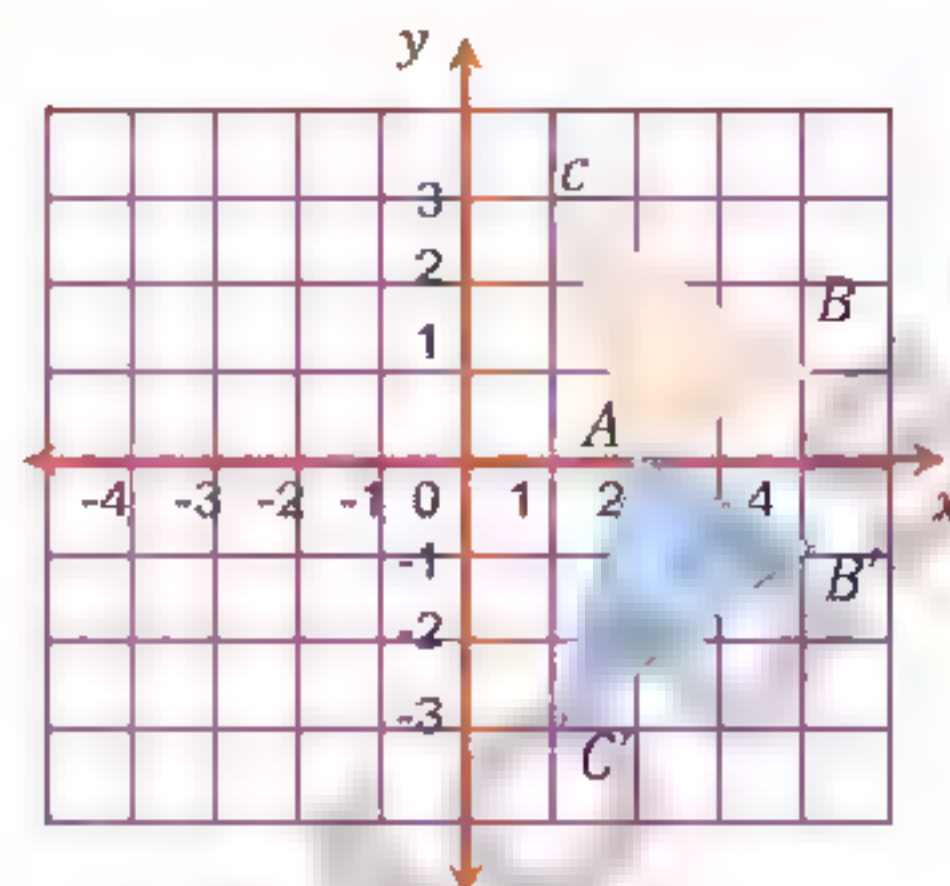
1. The image of the point (2, 3) under reflection over the X-axis is the point (2, -3).
2. The image of the point (-4, 1) under reflection over the Y-axis is the point (4, 1).

Draw the triangle with vertices A(2, 0), B(4, 1), and C(1, 3). Then draw its image under reflection over the X-axis:

$A(2,0) \xrightarrow[x-axis]{\text{Reflection}} A(2,0)$

1

$B(4,1) \xrightarrow[x-axis]{\text{Reflection}} B'(4,-1)$

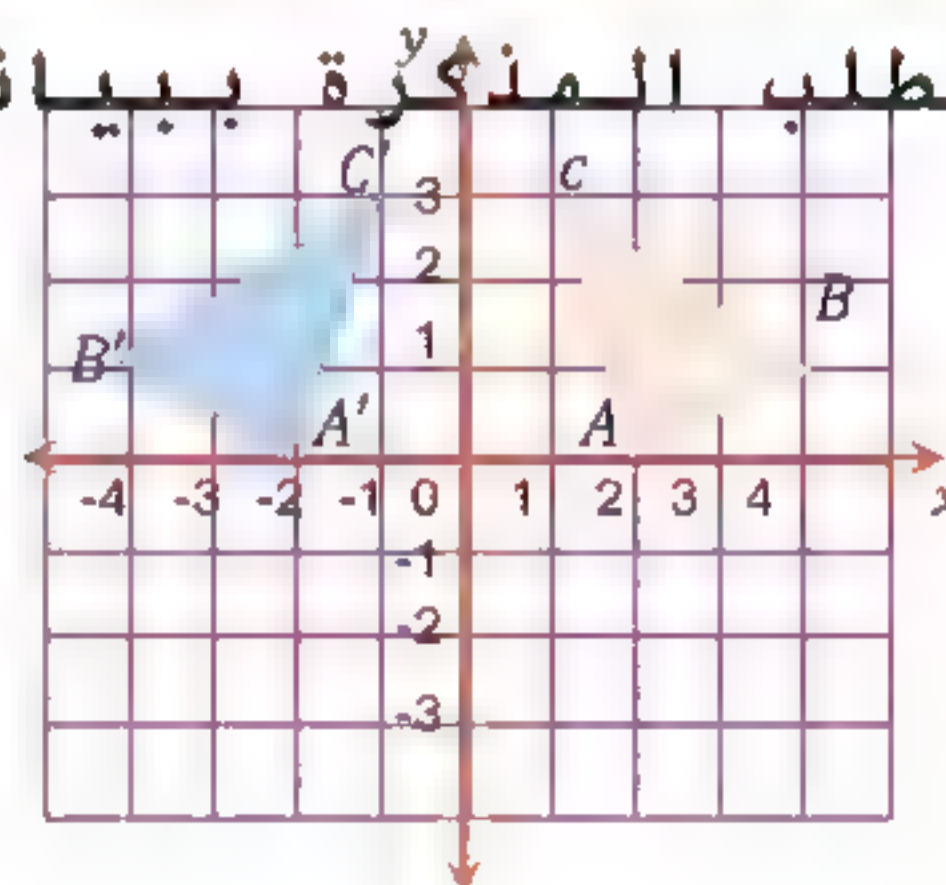
$C(1,3) \xrightarrow[x-axis]{\text{Reflection}} C'(1,-3)$

The triangle ΔAB'C' is the image of ΔABC under reflection over the X-axis.

Draw the triangle with vertices A(2, 0), B(4, 1), and C(1, 3). Then draw its image under reflection over the Y-axis:

$A'(2,0) \xrightarrow[y-axis]{\text{Reflection}} A'(-2,0)$

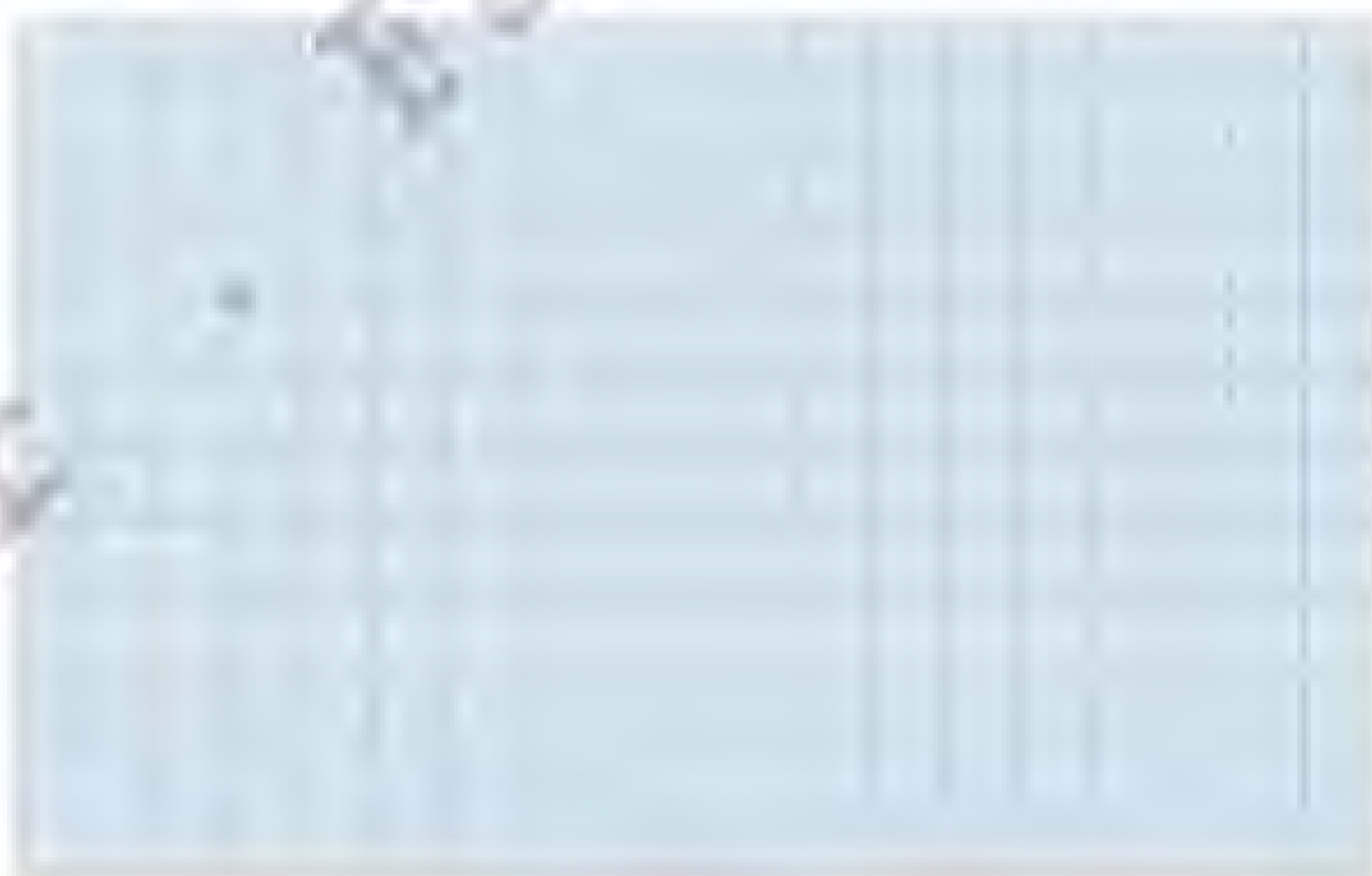
2

$B(4,1) \xrightarrow[y-axis]{\text{Reflection}} B'(4,-1)$

$C(1,3) \xrightarrow[y-axis]{\text{Reflection}} C'(1,-3)$

The triangle ΔA'B'C' is the image of ΔABC under reflection over the Y-axis.

1	The number of axes of symmetry of : (a) The equilateral triangle is .... (b) The isosceles triangle is.............. (c) The scalene triangle is .... (d) The parallelogram is.............. (e) The rectangle is .... (f) The rhombus is .... (h) The trapezium which is not isosceles is .... (g) The square is ...... (i) The isosceles trapezium .... (j) The circle ....
2	The reflection in a plane reserves: 1- .... 2- .... 3- .... 4- ....
3	If the reflection in a straight line transforms the figure to itself then this straight line is called ....
4	The image of the point $(1,3)$ by reflection in the X-axis is ....
5	The image of the point $(-2,5)$ by reflection in the y-axis is ....
6	The image of the point $(2,-3)$ by reflection in the .... is $(2,3)$
7	The image of the point $(-1,-4)$ by reflection in the .... is $(1,-4)$
8	The image of the point $(0,3)$ by reflection in the .... is itself.
9	The image of the point $(-5,0)$ by reflection in the .... is itself.
10	The image of the point $(2,1)$ by reflection in the X-axis followed by reflection in the y-axis is ....
11	The image of the point $(2,-3)$ by reflection in the y-axis followed by reflection in the X-axis is ....
12	$(-3,2)$ is the image of the point $(3,2)$ by reflection in ... ... axis

1. The image of the point $(-3,2)$ by reflection in the origin point is …….
   - $(3,2)$
   - $(-3,-2)$
   - $(3,-2)$
   - $(-3,2)$
2. The point $(5,-2)$ is the image of the point by reflection in the origin point ..
   - $(5,-2)$
   - $(-5,-2)$
   - $(-5,2)$
   - $(5,2)$
3. The point whose image by reflection in the origin point is itself is ……….
   - $(0,1)$
   - $(1,0)$
   - $(0,0)$
   - $(-1,0)$
4. The image of the point $(3,-2)$ by reflection in the origin point followed by reflection in x-axis is …….
   - $(3,-2)$
   - $(-3,-2)$
   - $(-3,2)$
   - $(3,2)$
5. The image of the point $(2,-5)$ by reflection in x-axis is ……
   - $(2,-5)$
   - $(2,5)$
   - $(-2,-5)$
   - $(5,2)$
6. The image of the point $(3,-5)$ by reflection in y-axis is ……
   - $(3,5)$
   - $(-3,-5)$
   - $(-3,5)$
   - $(-5,3)$

Find the image of $\triangle ABC$ where $A(-6,-1)$, $B(-2,-1)$ and $C(-5,-6)$ by reflection in the x-axis

……………………………………………………………………………………………

……………………………………………………………………………………………

1

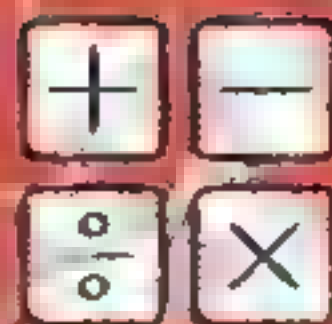
In the XY-coordinate plane, draw △ABC, where A(-1, 5), B(6, 1), and C(4, -2), then find the image of △ABC under reflection through the origin.

....

2

In the XY-coordinate plane, draw △ABC, where A(-1, 5), B(6, 1), and C(4, -2), then find the image of △ABC under reflection through the origin.

....

3

Exercises (3)

1	If the reflection in a straight line transforms the shape into itself, this line is called ....
2	The image of the point (2, 4) under reflection over the x-axis is ....
3	The image of the point (8, -1) under reflection over the y-axis is ....
4	The image of the point (-3, 2) under reflection in .... is (2, -3).
5	The image of the point (4, 1) under reflection in .... is (1, -4).
6	The image of the point (0, 5) under reflection in .... is itself.
7	The image of the point (0, -9) under reflection in .... is itself.
8	The image of the point (3, 2) under reflection over the x-axis followed by reflection over the y-axis is ....
9	The image of the point (-2, 1) under reflection over the y-axis followed by reflection over the x-axis is ....

The image of the point (-3, 2) by reflection in the origin point is: ....

(3, 2)		(-4, -3)		(4, -3)		(-4, 3)

The point (6, -3) is the image of the point by reflection in the origin point: ......

(6, -3)		(-6, -3)		(-6, 3)		(6, 3)

The point whose image by reflection in the origin point is itself is: ....

(0, 2)	(b)	(2, 0)	(c)	(0,0)	(d)	(-2, 0)

The image of the point (4, -3) by reflection in the origin point followed by reflection in the x-axis is: ....

(4, -3)	(b)	(-4, -3)	(c)	(-4, 3)	(d)	(4, 3)

The image of the point (1, -4) by reflection in the x-axis is:

(1, -4)	(b)	(1, 4)	(c)	(-1, -4)	(d)	(4, 1)

The image of the point (5, -6) by reflection in the y-axis is: ....

(5, 6)	(b)	(-5, -6)	(c)	(−5, 6)	(d)	(-6, 5)

$F(5-6) \to F'(-5,-6)$	$M(7,-9) \to M'(7,9)$ ....
$N(-3,0) \to N'(-3,0)$	$C(0,5) \to C'(0,5)$ ....

Example 4

1

ABCD is a rectangle with vertices A(2, 5), B(6, 5), C(6, 8), and D(2, 8). Find the image of rectangle ABCD under reflection in the origin:

....

2

Draw the square ABCD with vertices A(0, 2), B(-5, 0), C(-3, -5), and D(2, -3). Then draw its image under reflection over the x-axis. Compare the lengths of the sides and the area of the square:

....

Translation

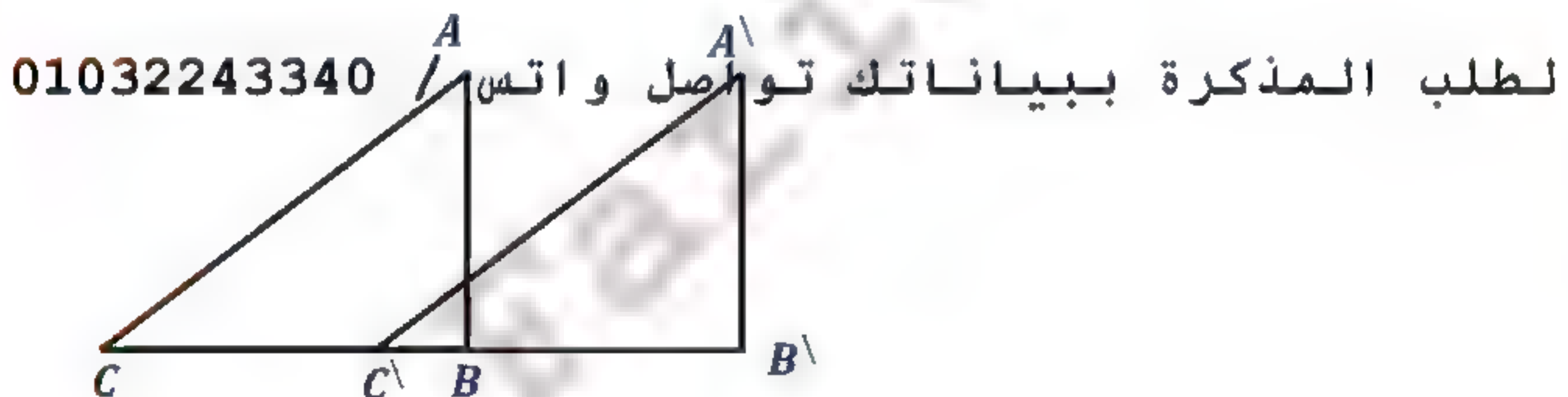
Translation is a geometric: transformation that moves every point in the plane by a fixed distance in a specific direction.

The translation is determined by:

1. The magnitude of the translation.
2 - The direction of the translation.

Properties of Translation:

1. It preserves the lengths of line segments.
2. It preserves the measures of angles.
3. It preserves parallelism.
4. It preserves collinearity.
5. It preserves the rotational order of the vertices of the shape.

-

Important Notes:

In the coordinate plane, translation can be expressed as: $A + T = A'$

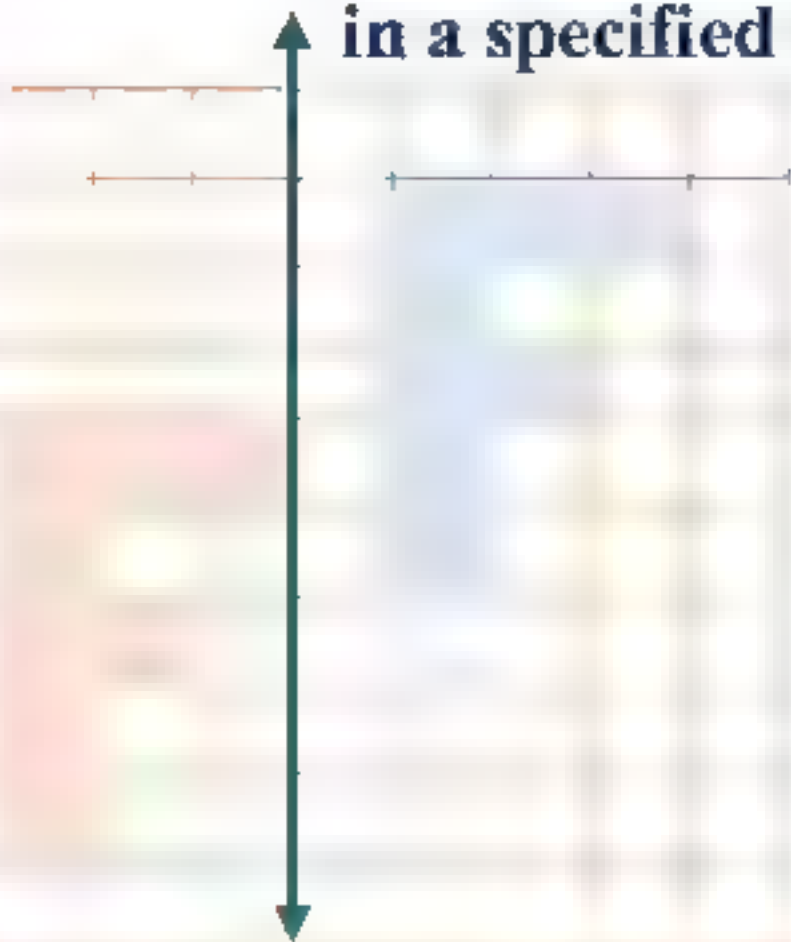
(Where A is the original point, T is the translation vector, and A' is the image after translation.)

To find the translation from the original to the image: $A' - T = A$

And to find the translation vector: $A' - A = T$

(Where A' is the translated image and A is the original point.)

Translation

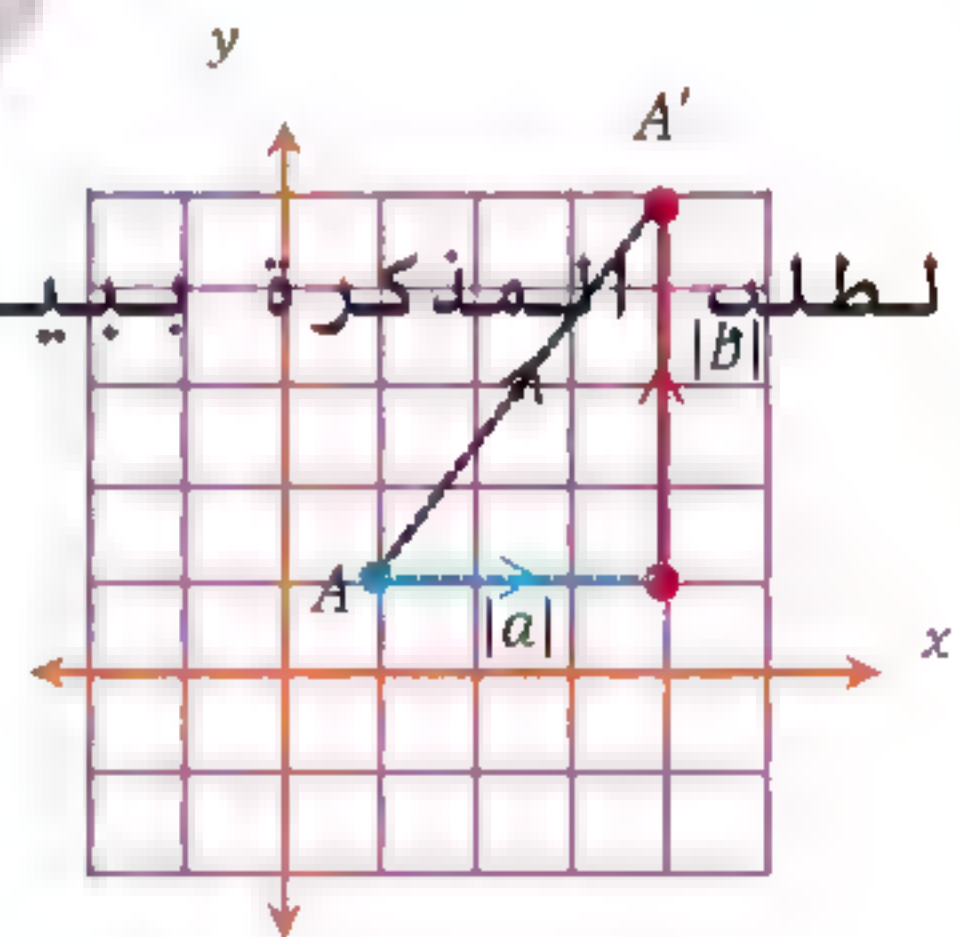
Translation: It is the shifting of a shape along a straight line by a specified distance in a specified direction.

Second: Translation in the coordinate plane.

Translation in the coordinate plane is determined by the horizontal shift a and the vertical shift b.

This is expressed as the ordered pair (a, b).

The image of point A(x, y) under the translation (a, b) is the point A'(x + a, y + b).

For example:

The image of the point (3, 1) under a translation of 3 units to the right and 4 units up (translation (3, 4)) is: A'(3 + 3, 1 + 4), which is A'(6, 5).

1. The image of the point (3, 1) under a translation of 3 units to the right and 4 units up (translation (3, 4)) is the point (3 + 3, 1 + 4), which is A'(6, 5).
2. The image of the point (2, 3) under a translation (4, 5) is (2 + 4, 3 + 5), which is (6, 8).
3. The image of the point (5, 9) under a translation (Y - 1, X + 2) is (5 + 2, 9 - 1) , which is (7, 8).

4 **If the point A(-3, 5) is the image of point B under a translation (2, -1), then point B is (-3 - 2, 5 + 1), which is (-5, 6).**

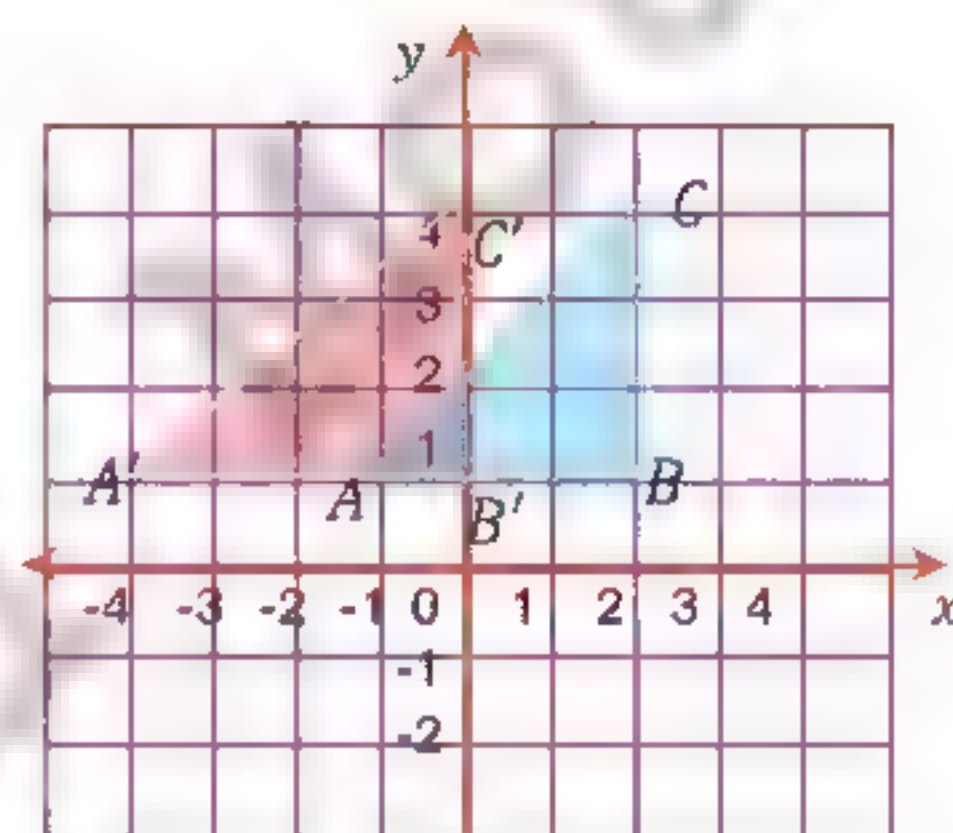
5 **The point (..... , ...) under the translation (Y + 4, X - 3) becomes (5, -11). Solving gives the original point as (-8, 2).**

Draw triangle ABC with vertices C(3, 4), B(3, 1), and A(-1, 1), then find its image under a translation of 3 units to the left (equivalent to translation (-3, 0)):

1

$$A(-1\,,1)\xrightarrow[(-3\,,0)]{\text{translation}}A(-4\,,1)$$

$$B(3\,,1)\xrightarrow[(-3\,,0)]{\text{translation}}B'(0\,,1)$$

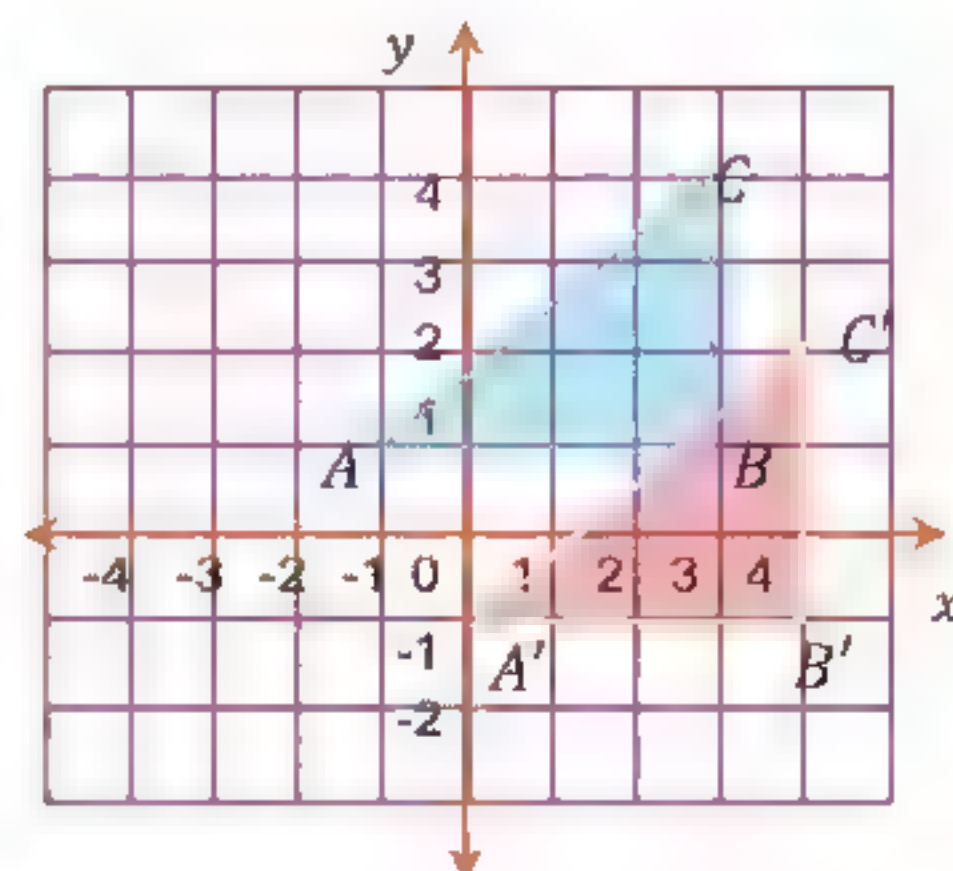
$$C(3,4)\xrightarrow[(-3\,,0)]{\text{translation}}C'(0\,,4)$$

The triangle ΔA'B'C' is the image of ΔABC under the translation of 3 units to the left.

Draw triangle ABC with vertices C(3, 4), B(3, 1), and A(-1, 1), then find its image under a translation (1, -2):

2

$$A(-1\,,1)\xrightarrow[(1\,,-2)]{\text{translation}}A'(0\,,-1)$$

$$B(3\,,1)\xrightarrow[(1\,,-2)]{\text{translation}}B'(4\,,-1)$$

$$C(3,4)\xrightarrow[(1\,,-2)]{\text{translation}}C'(4\,,2)$$

The triangle ΔA'B'C' is the image of ΔABC under the translation (1, -2).

3 Find the images of points C(4, 1), B(-2, 2), and A(-6, 6) under the translation (x, y) → (x - 1, y + 3):

....

....

....

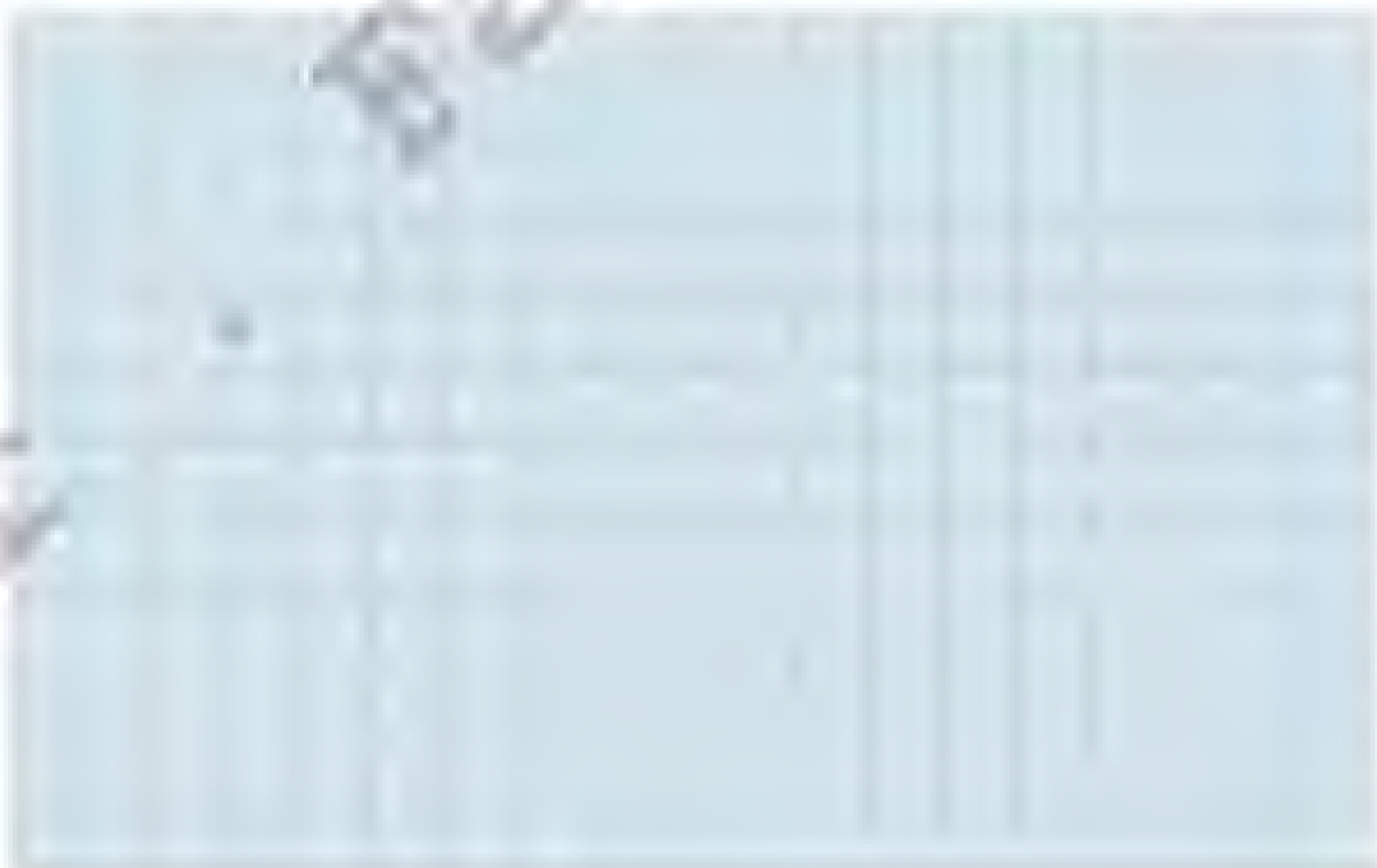
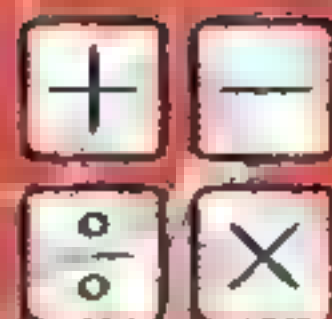
4

Draw the segment AB where A(3, 3) and B(1, 0), then draw its image under the translation (Y - 2, X + 1):

....

5

In an orthogonal coordinate plane, locate the points A(5, 5), B(5, [illegible]), and C([illegible]), then draw the image of the triangle under the translation (-4, -2):

....

Exercises(4)

The image of the point $(1,5)$ by translation $(1,3)$ is ………………	The image of the point $(5,3)$ by translation $(-4,5)$ is ………………
The image of the point $(1,5)$ by translation $(x-1,y)$ is …………	The image of the point $(5,9)$ by translation $(x+1,y-2)$ is ………
The image of the point $(3,-4)$ by translation of 7 units in the positive direction of the x-axis is …………	The image of the point $(3,2)$ by translation of 3 units in the negative direction of the y-axis is ………

1. **The image of the point $(3,2)$ by translation $(x,y) \longrightarrow (x+3,y-2)$ is ……………**
2. **The image of the point $(-5,4)$ by translation $(x,y) \longrightarrow (x+4,y-5)$ is …………**
3. **The image of the point $(-2,-5)$ by translation $(x,y) \longrightarrow (x-2,y)$ is …………**
4. **The image of the point $(3,-2)$ by translation $(x,y) \longrightarrow (x,y+3)$ is …………**

The image of the point $(-1,2)$ by translation of magnitude of 3 units in the positive direction of the X-axis is ………

(−1,5)	(2,2)	(−2,2)	(−1,3)

The image of the point $(-3,4)$ by translation of magnitude of 4 units in the negative direction of the y-axis is …….

(−3,0)	(−7,4)	(−3,8)	(−1,4)

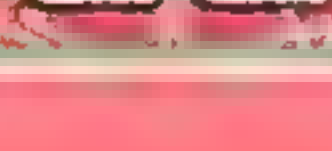
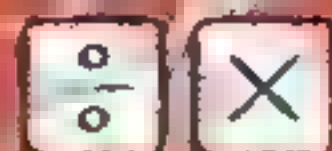
If $A'(3,-3)$ is the image of A by translation $(x,y) \longrightarrow (x-1,y-4)$, then the point A is ………..

(2,−7)	(4,1)	(−4,−1)	(2,1)

The image of the point $(-1,4)$ by the translation $(3,-2)$ followed by reflection in the X-axis is ……….

(2,2)	(−2,2)	(−2,−2)	(2,−2)

If the point $(a,-1)$ is the image of $(2,4)$ by the translation $(x,y) \longrightarrow (x+1,y-b)$, then (a,b) is ……….

(3,3)	(1,3)	(3,5)	(1,−5)

The square has vertices A(1, 1), B(4, 2), C(3, 5), and D(0, 4). Draw the square and its image under the translation (1, -1):

....

....

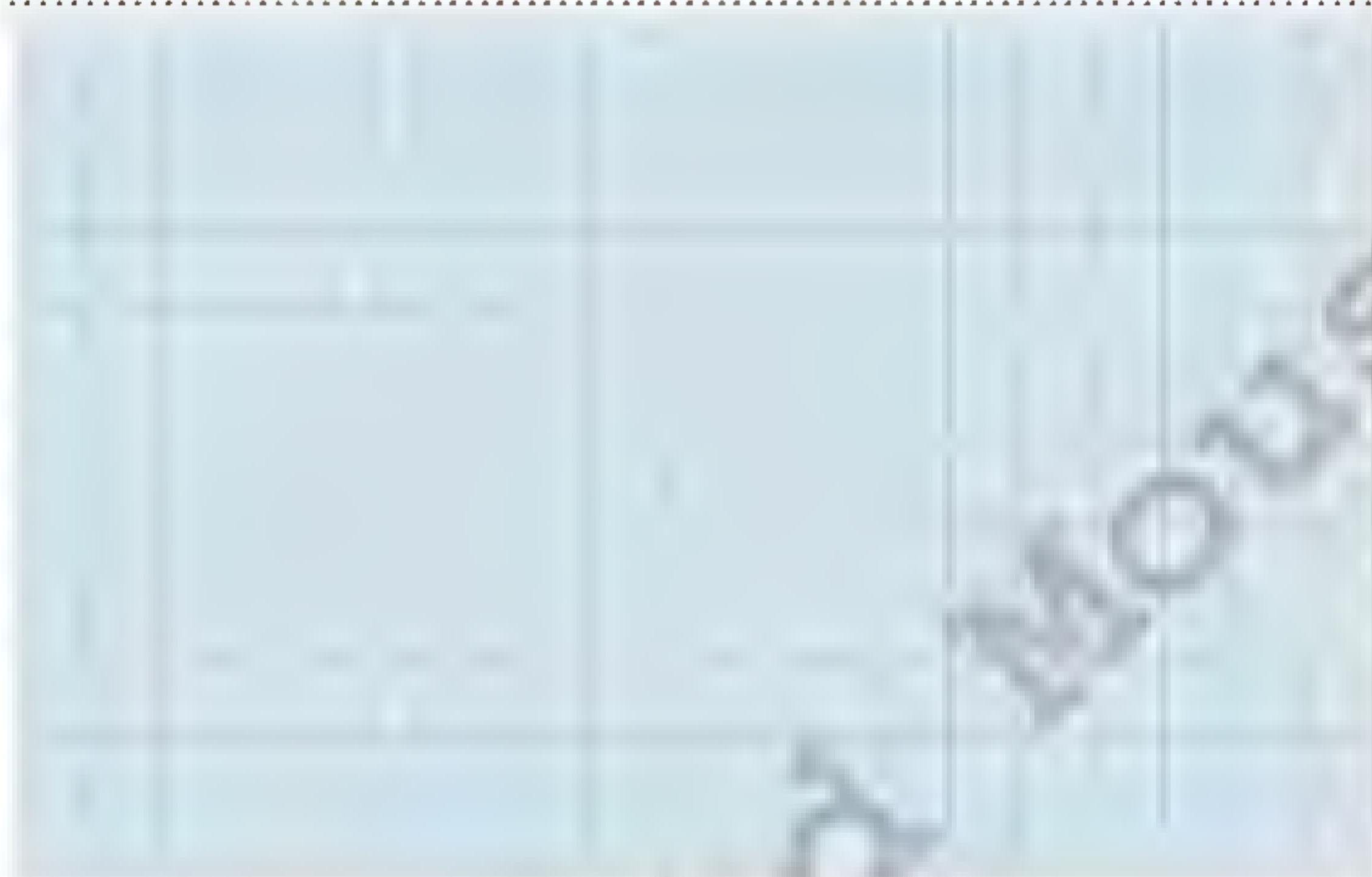
1

If the image of point A(1, 1) under a translation is A'(2, 2), find the images of points O(0, 0), B(-1, 3), and C(-3, 5) using the same translation

....

2

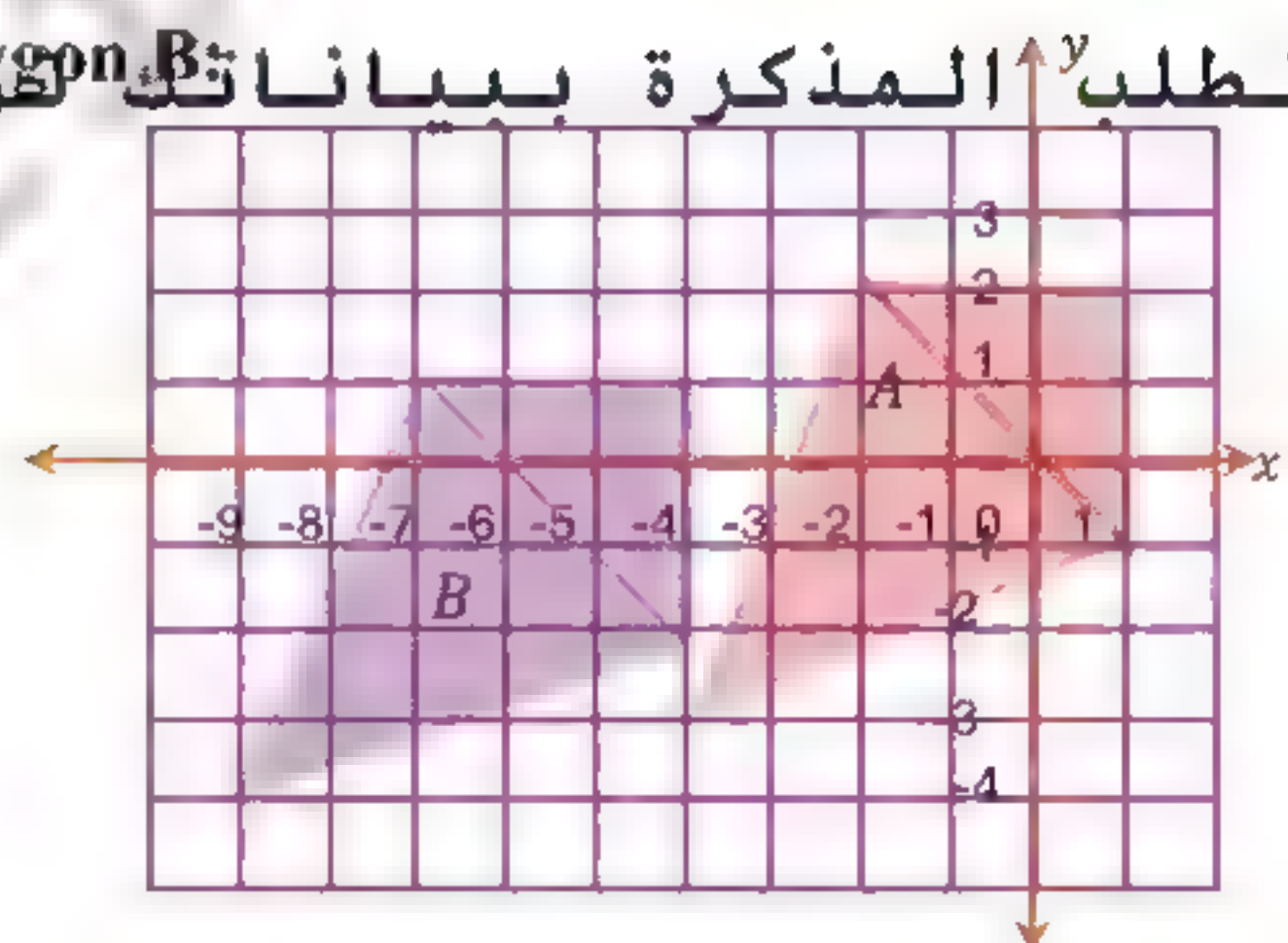
3. Draw triangle ΔABC where C(-2, 3), B(-1, 1), A(-4, 1), then draw its image under the translation (2, 1)

....

4. In the following diagram, identify the translation that makes polygon A the image of polygon B:

....

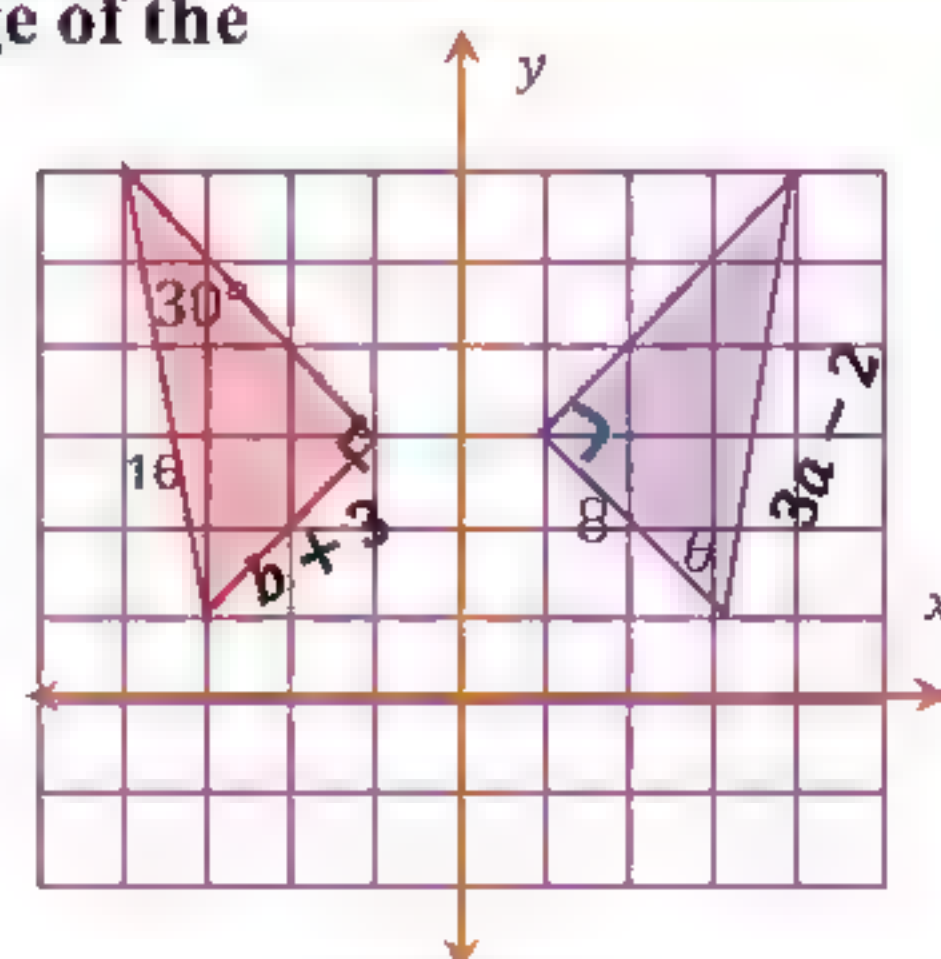
....

....

....

5. In the following diagram, if one triangle is the image of the other under reflection in the y-axis, find the values of θ, b, and a

....

....

....

rotation

Rotation is determined by:

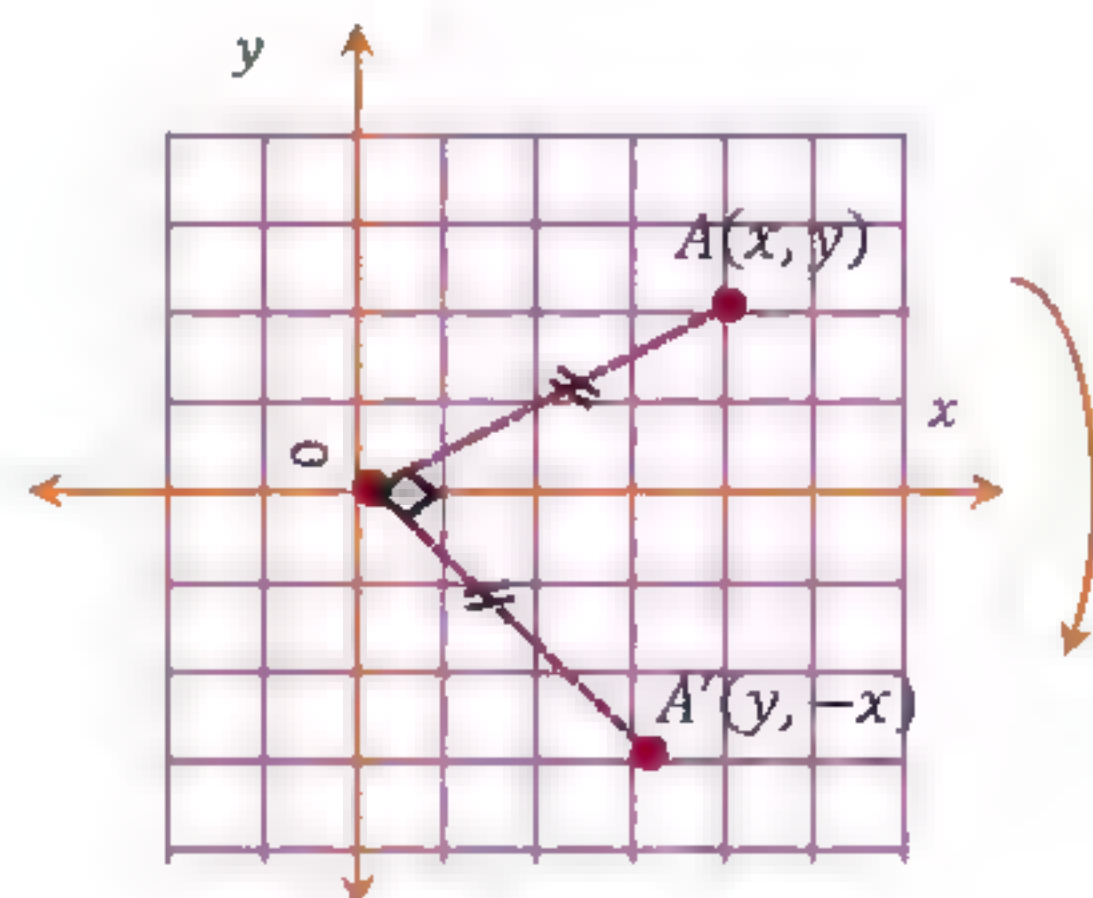
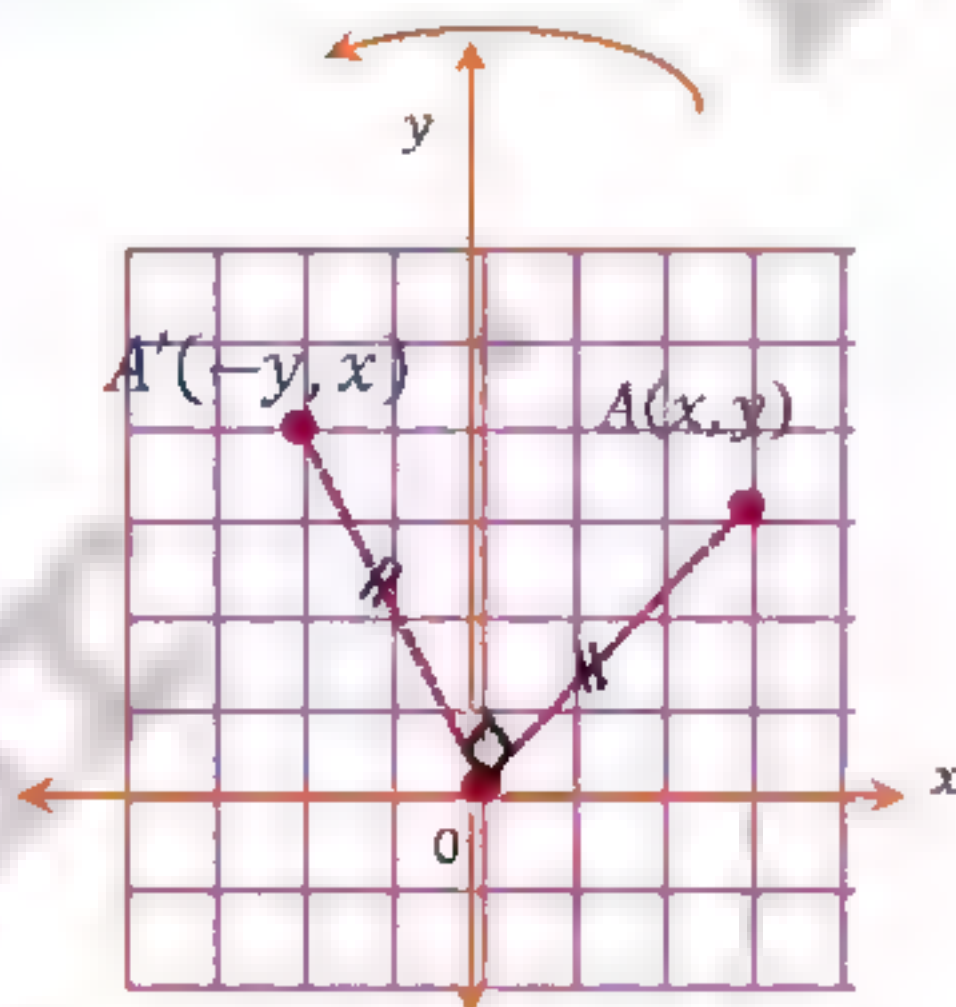
1. The center of rotation 2 - The angle of rotation 3 - The direction of rotation

- Positive rotation is counterclockwise (opposite to the direction of the clock's hands).
- Negative rotation is clockwise (in the same direction as the clock's hands).

Properties of Rotation:

1. It preserves the lengths of line segments.
2. It preserves the measures of angles.
3. It preserves parallelism.
4. It preserves collinearity (points on the same line remain on the same line).
5. It preserves the rotational order of the vertices of the shape (the relative order of points does not change).

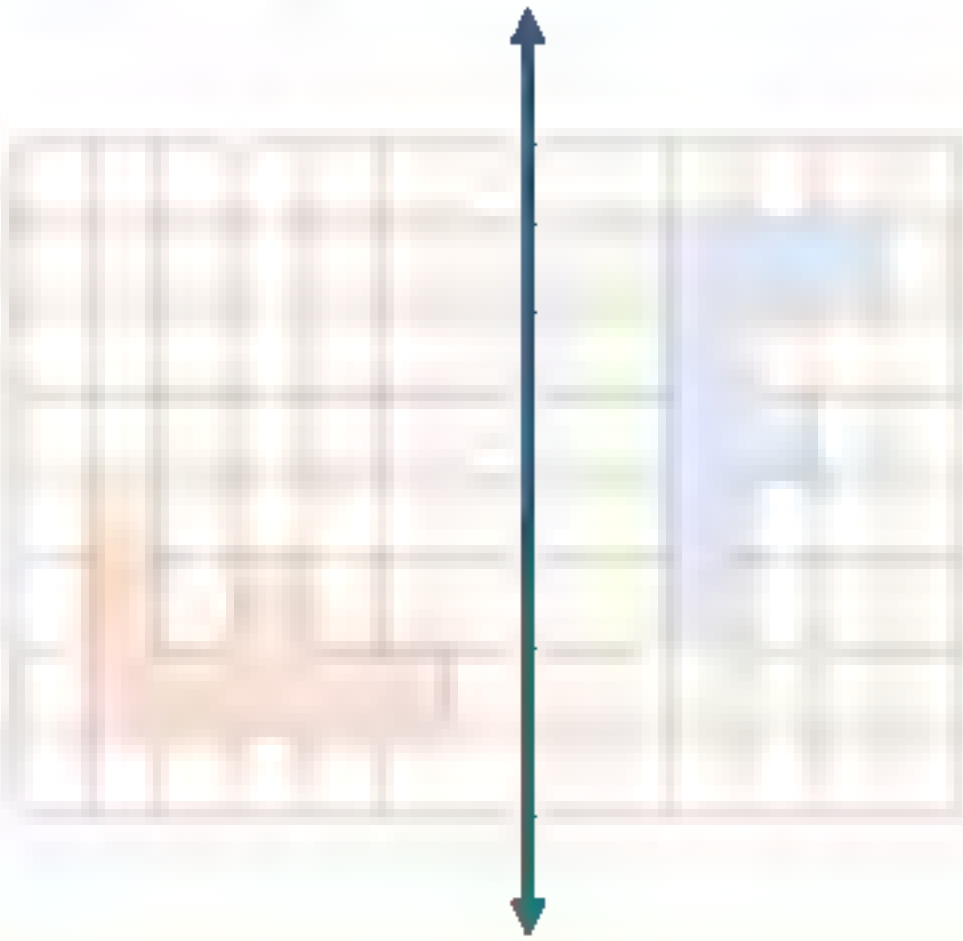
Types of Rotation:

1. ±360° rotation is called a full rotation (or complete turn), where the point returns to its original position.
2. ±180° rotation is called a half-turn and is equivalent to a reflection at the origin.
3. ±90° rotation is called a quarter-turn.
4. $A(x,y) \xrightarrow{R(0,90^\circ)} A'(-y,x)$ $A(x,y) \xrightarrow{R(0,-90^\circ)} A'(y,-x)$

(5)**Note that:**

- Rotation R(0, 90°) is equivalent to rotation R(0, -270°).
- Rotation R(0, 270°) is equivalent to rotation R(0, -90°).

(6) $A(x,y) \xrightarrow{R(0,+180^\circ)} A'(-x,-y)$

Third: Rotation in the coordinate plane.

It is the turning of a shape around a point called the center of rotation, by a specific angle and in a specific direction.

Example 1

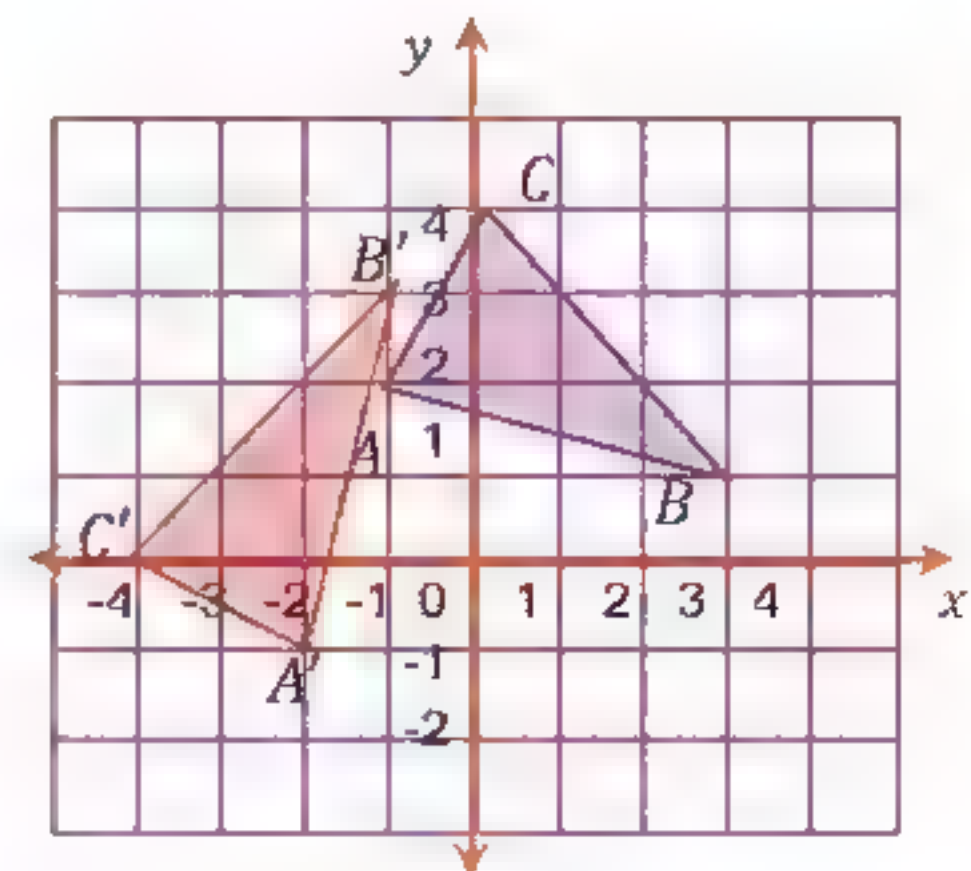
1. The image of the point (3, 1) under the rotation R(0, 90°) is the point (-1, 3).
2. The image of the point (-3, 4) under the rotation R(0, -90°) is the point (4, 3).

3. The image of the point (-1, 5) under the rotation R(0, -270°) is the point ....
4. The image of the point (2, -5) under the rotation R(0, 180°) is the point ....

Example 2

Draw triangle ABC on the coordinate plane where A(-1, 2), B(3, 1), C(0, 4). Then draw its image under the rotation R(0, 90°):

$A(-1, 2) \xrightarrow{R(0, 90°)} A'(-2, -1)$

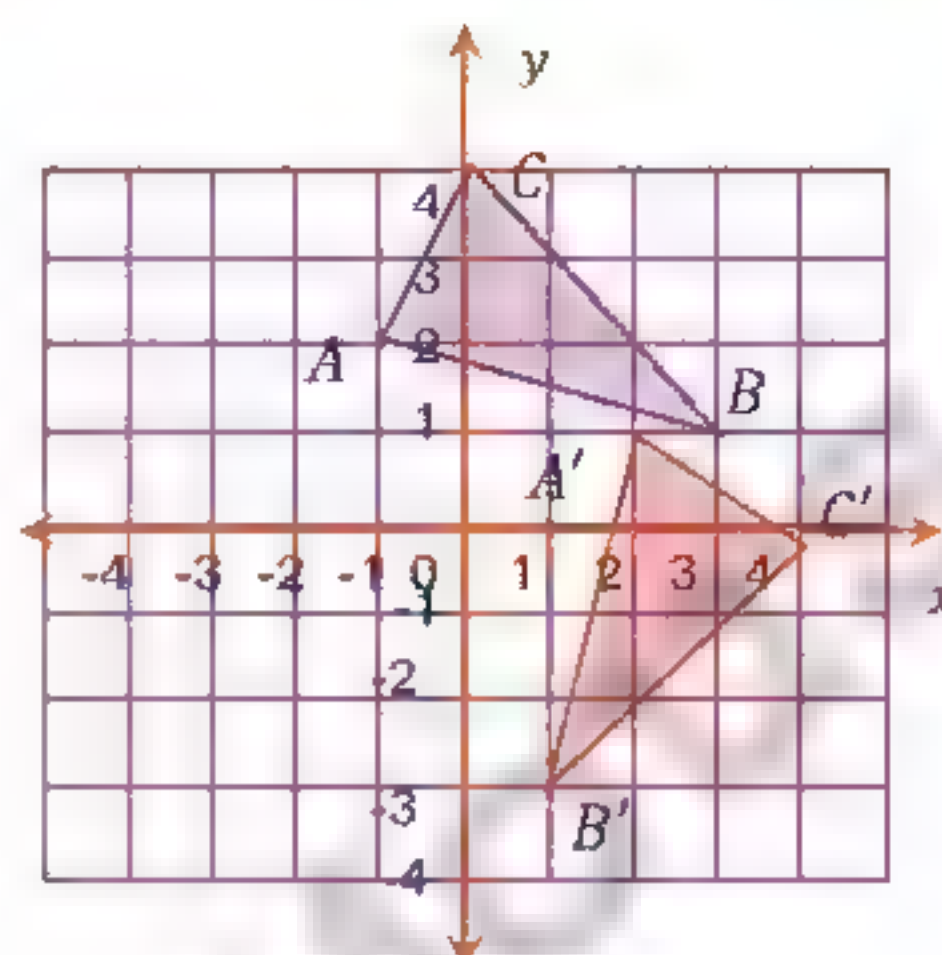
$B(3, 1) \xrightarrow{R(0, 90°)} B'(-1, 3)$

1 $C(0, 4) \xrightarrow{R(0, 90°)} C'(-4, 0)$

The triangle ΔA'B'C' is the image of ΔABC under the rotation R(0, 90°).

Draw triangle ABC on the coordinate plane where A(-1, 2), B(3, 1), C(0, 4). Then draw its image under the rotation R(0, -90°):

$A(-1,2) \xrightarrow{R(0,-90^\circ)} A'(2,1)$

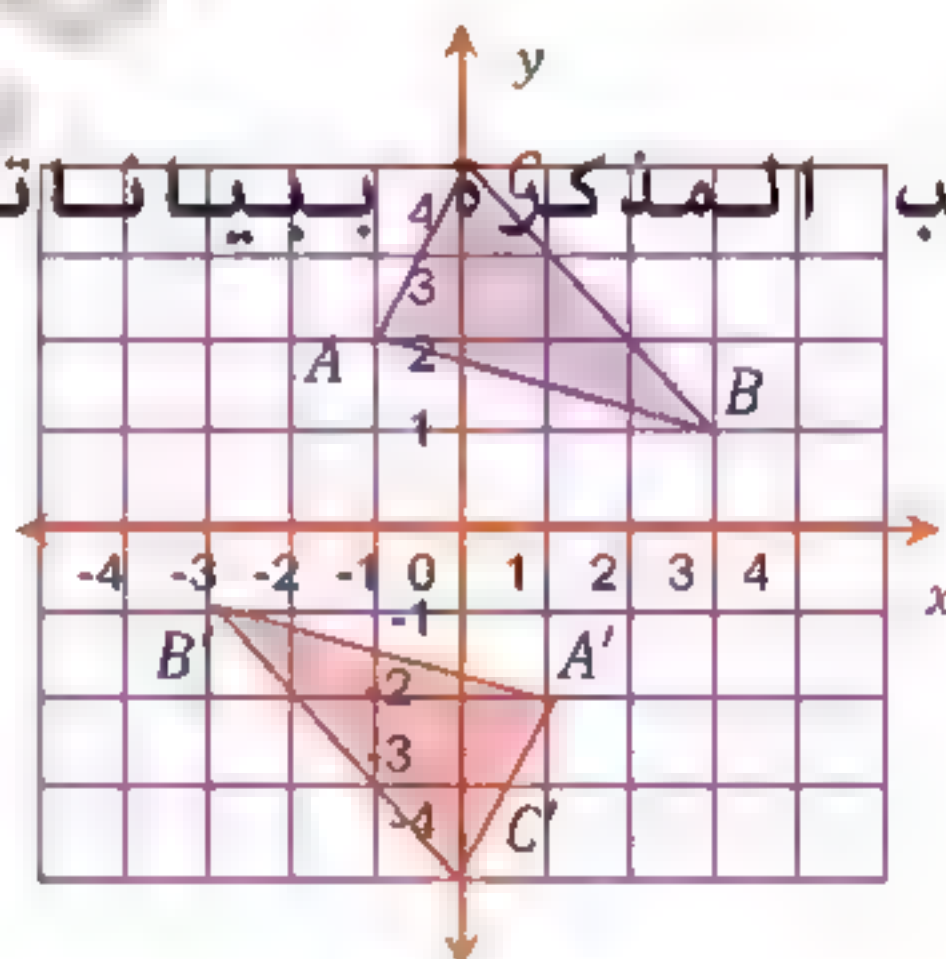
$B(3,1) \xrightarrow{R(0,-90^\circ)} B'(1,-3)$

2 $C(0,4) \xrightarrow{R(0,-90^\circ)} C'(4,0)$

The triangle ΔA'B'C' is the image of ΔABC under the rotation R(0, -90°)

Draw triangle ABC on the coordinate plane where A(-1, 2), B(3, 1), C(0, 4). Then draw its image under the rotation R(0, 180°):

$A(-1,2) \xrightarrow{R(0,180^\circ)} A'(1,-2)$

$B(3,1) \xrightarrow{R(0,180^\circ)} B'(-3,-1)$

3 $C(0,4) \xrightarrow{R(0,180^\circ)} C'(0,-4)$

The triangle ΔA'B'C' is the image of ΔABC under the rotation R(0, 180°).

Example 1

1. The image of the point $(3,-5)$ by rotation about the origin point with an angle of measure 90° is .... and with an angle of measure 180° is ....
2. The image of the point $(7,0)$ by rotation about the origin point with an angle of measure 90° is .... and with an angle of measure 360° is ....
3. The point $(2,-4)$ is the image of the point $(1,4)$ by rotation about the origin point with an angle of measure ....

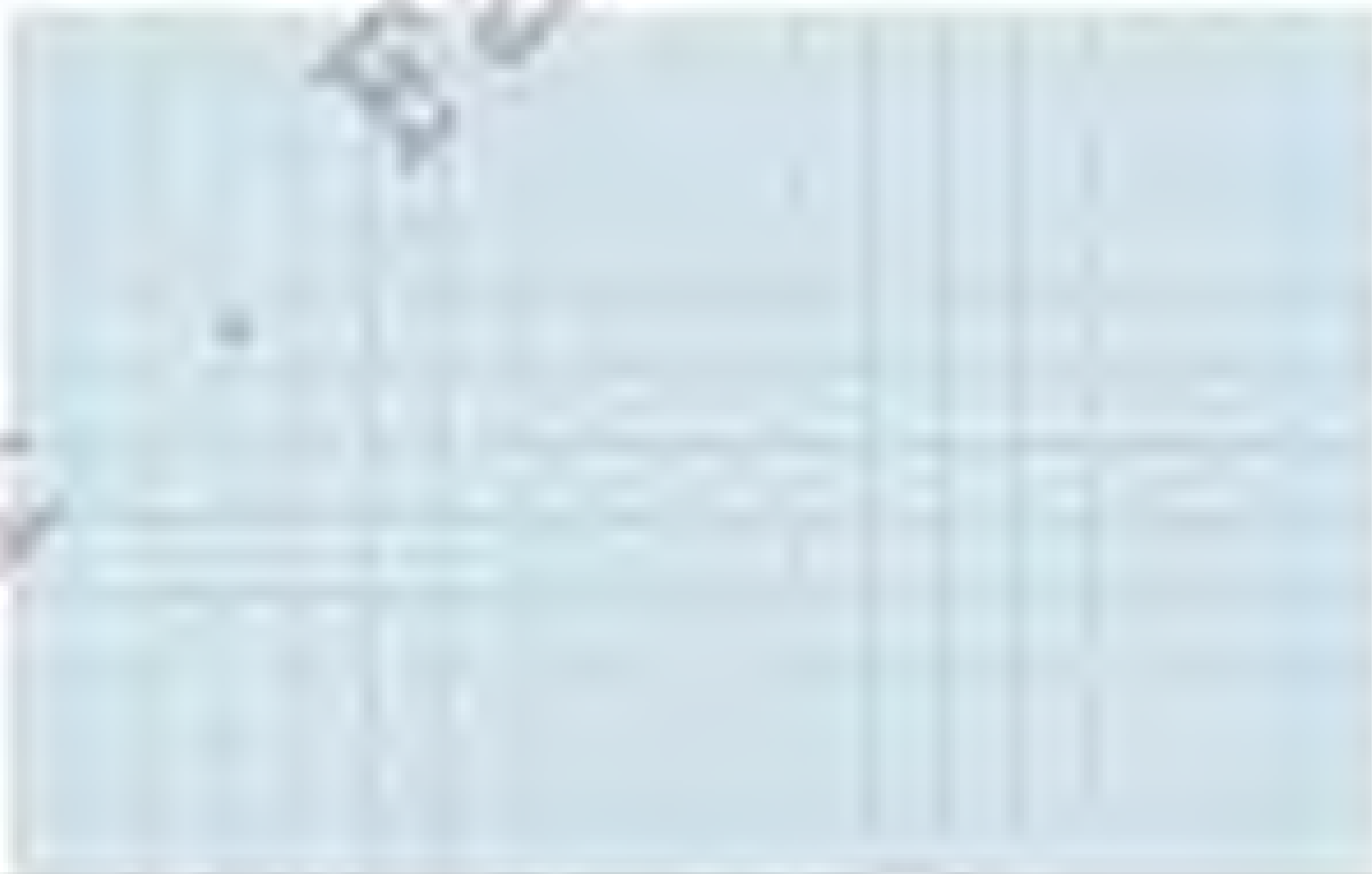
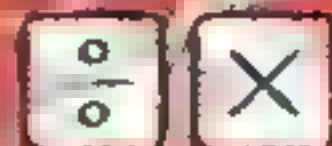
4. The image of the point .... by rotation about the origin point with an angle of measure $90°$ is $(-2,8)$
5. The image of the point .... by rotation about the origin point with an angle of measure $(-180°)$ is $(3,-1)$
6. The image of the point $(-7,2)$ by rotation $90°$ about the origin point followed by reflection in y-axis is ....
7. The image of the point $(-1,0)$ by translation $(x,y) \longrightarrow (x+2, y-4)$ followed by rotation about the origin point with an angle of measure $90°$ is ....
8. The rotation with an angle of measure $90°$ about the origin point maps the point $(x,-y)$ onto the point ....
9. The image of (a,b) is the same point by rotation about the origin point with an angle of measure ....

Example 4

On the coordinate grid, draw the quadrilateral ABCD with the points:
A(0, 4), B(4, 4), C(7, 0), D(0, 0).

Then draw its image resulting from the rotation around the origin O by an angle of R(O, -180°).

....

1

The points A(2, 1) and B(1, 3). Draw the image of the line segment AB resulting from the rotation around the origin by an angle of 90° counterclockwise.

....

2

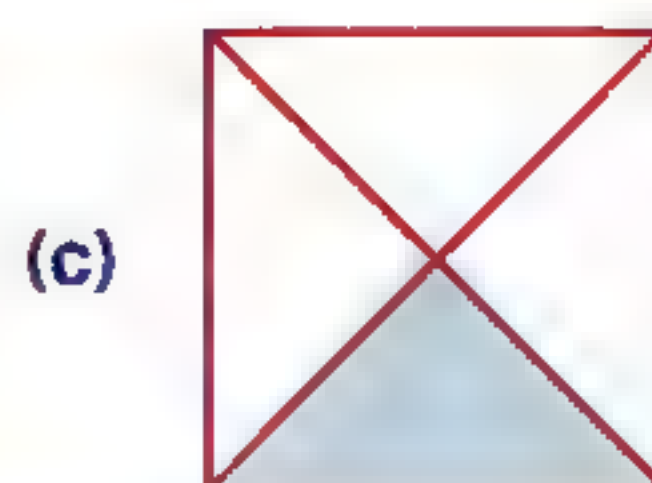
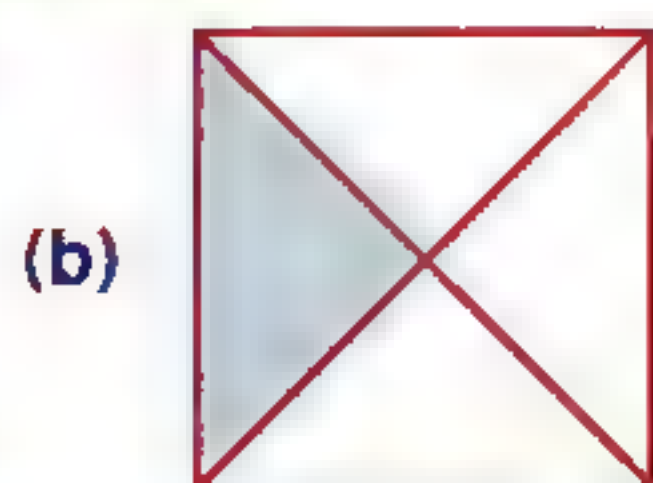
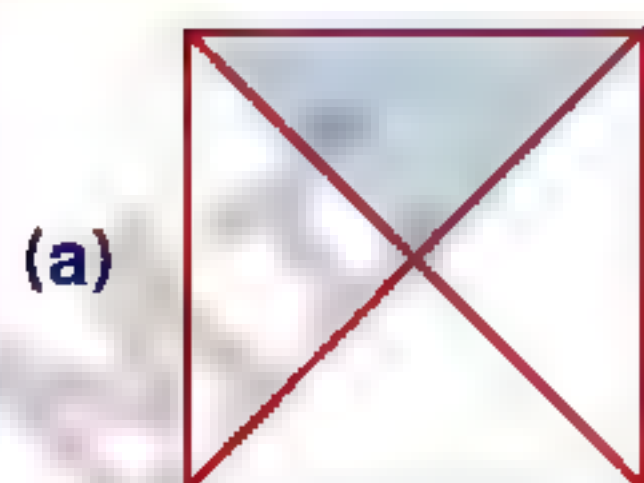
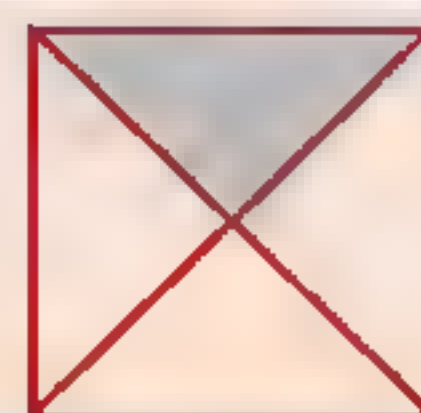
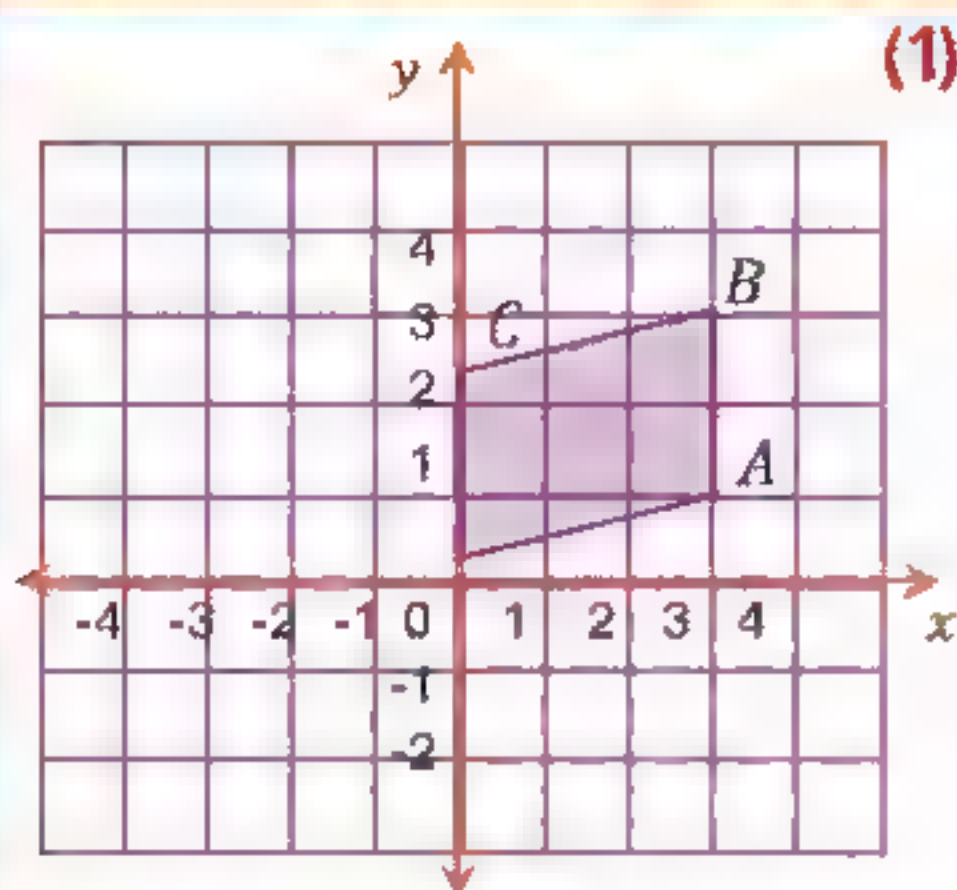
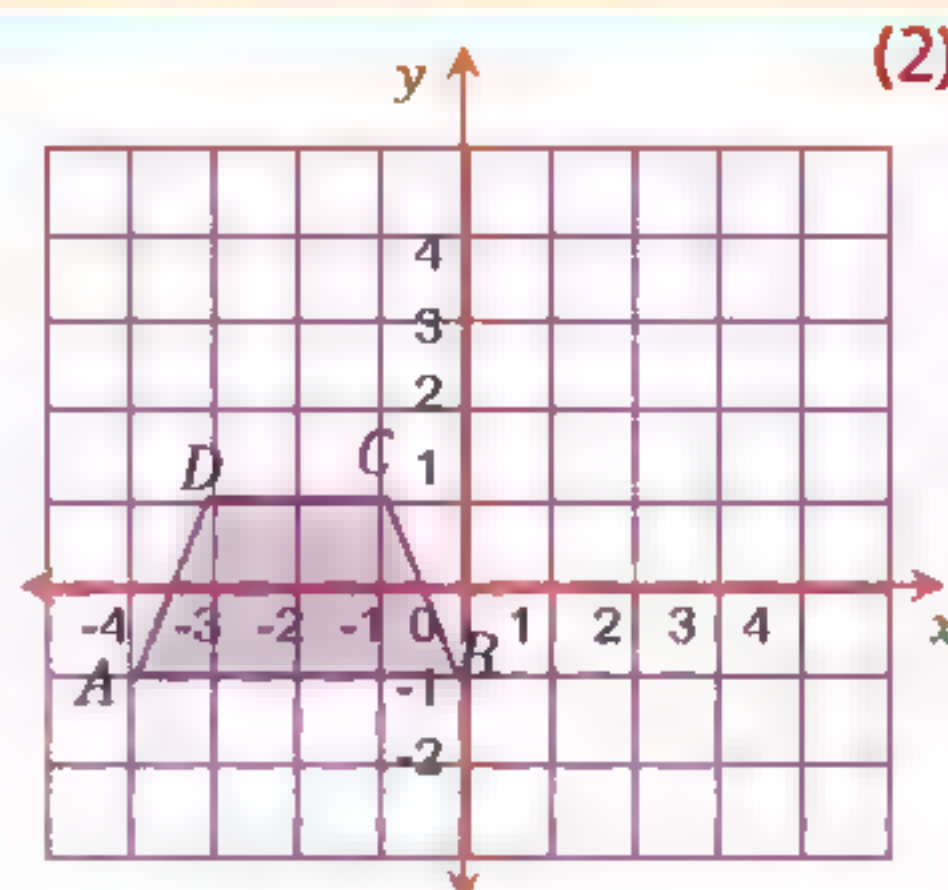
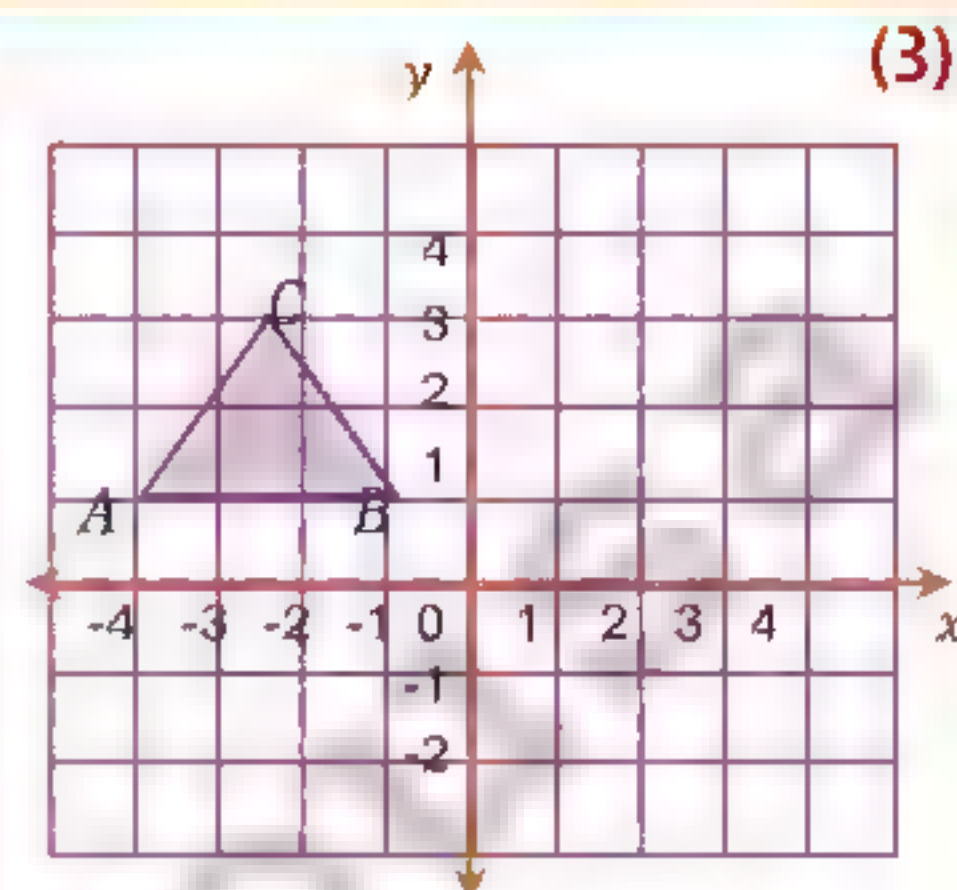
On graph paper, draw the triangle ΔABC where A(3, -1), B(5, 2), C(-2, 4).
Then draw its image resulting from the rotation around the origin by an angle of 180°.

....

3

Exercises (5)

Question 1: Choose the correct answer from the given options.

1	What is the image of the point (-4, 2) under a rotation around the origin O by an angle of 90° counterclockwise?						
(a)	(-4 , -2)	(b)	(4 , 2)	(c)	(-2 , 4)	(d)	(-2 , -4)
2	أي من الدورانات الآتية تجعل النقطة $A'(x\,,-y)$ صورة $A(-x\,,y)$؟						
(a)	$R(0\,,-90°)$	(b)	$R(0\,,90°)$	(c)	$R(0\,,180°)$	(d)	$R(0\,,360°)$
3	If A' is the image of point A under reflection over the X-axis, and point A is in the third quadrant, in which quadrant does point A' lie?						
(a)	First	(b)	Second	(c)	Third	(d)	Fourth
4	What is the image of the point (5, -2) after a translation of 5 units in the negative X-direction?						
(a)	(5 , -7)	(b)	(10 , -2)	(c)	(0 , -2)	(d)	(5 , -3)
5	What rotation makes the image of point A(2, -6) the point A'(-6, -2)?						
(a)	$R(0\,,-180°)$	(b)	$R(0\,,-90°)$	(c)	$R(0\,,90°)$	(d)	$R(0\,,180°)$
6	Which of the following represents a rotation of the given square around its center by an angle of 90° clockwise?						
(a)		(b)		(c)		(d)	
7	If the point A'(x + 1, -2) is the image of point A(-4, 2) under a rotation around the origin O by an angle of 180°, what is the value of x?						
(a)	3	(b)	-1	(c)	-2	(d)	-5

Reflection over the y-axis

Translation (3, 4)

Rotation around the origin O clockwise by an angle of 90°

1. The image of the point $(2, -3)$ by rotation about the origin point with an angle of measure 90° is .... and with an angle of measure 180° is ....

2. The image of the point $(-1, 5)$ by rotation about the origin point with an angle of measure 90° is .... and with an angle of measure 360° is ....
3. The point $(3, -2)$ is the image of the point $(2, 3)$ by rotation about the origin point with an angle of measure ....
4. The image of the point .... by rotation about the origin point with an angle of measure 90° is $(-1, 4)$
5. The image of the point .... by rotation about the origin point with an angle of measure (-180°) is $(5, -2)$
6. The image of the point $(-3, 7)$ by rotation 90° about the origin point followed by reflection in y-axis is ....
7. The image of the point $(-2, 0)$ by translation $(x, y) \longrightarrow (x + 3, y - 1)$ followed by rotation about the origin point with an angle of measure 90° is ....
8. The rotation with an angle of measure 90° about the origin point maps the point $(x, -y)$ onto the point ....
9. The image of (a, b) is the same point by rotation about the origin point with an angle of measure ....
10. If the image of the point (x, y) by rotation about the origin point with an angle of measure 90° is (a, b), then $a + y =$ ....

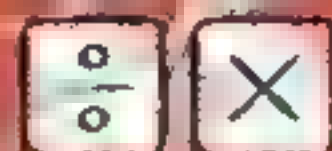
Draw triangle ABC where C(4, 1), B(-2, 2), A(-6, 6), then draw its image under the rotation R(0, -90°).

....

1

Draw triangle ABC where C(4, 1), B(-2, 2), A(-6, 6), then draw its image under the rotation R(0, 90°).

....

2

Find the image of quadrilateral ABCD where D(7, 1), C(9, -4), B(6, -3), A(4, -1) under the rotation R(0, -270°).

....

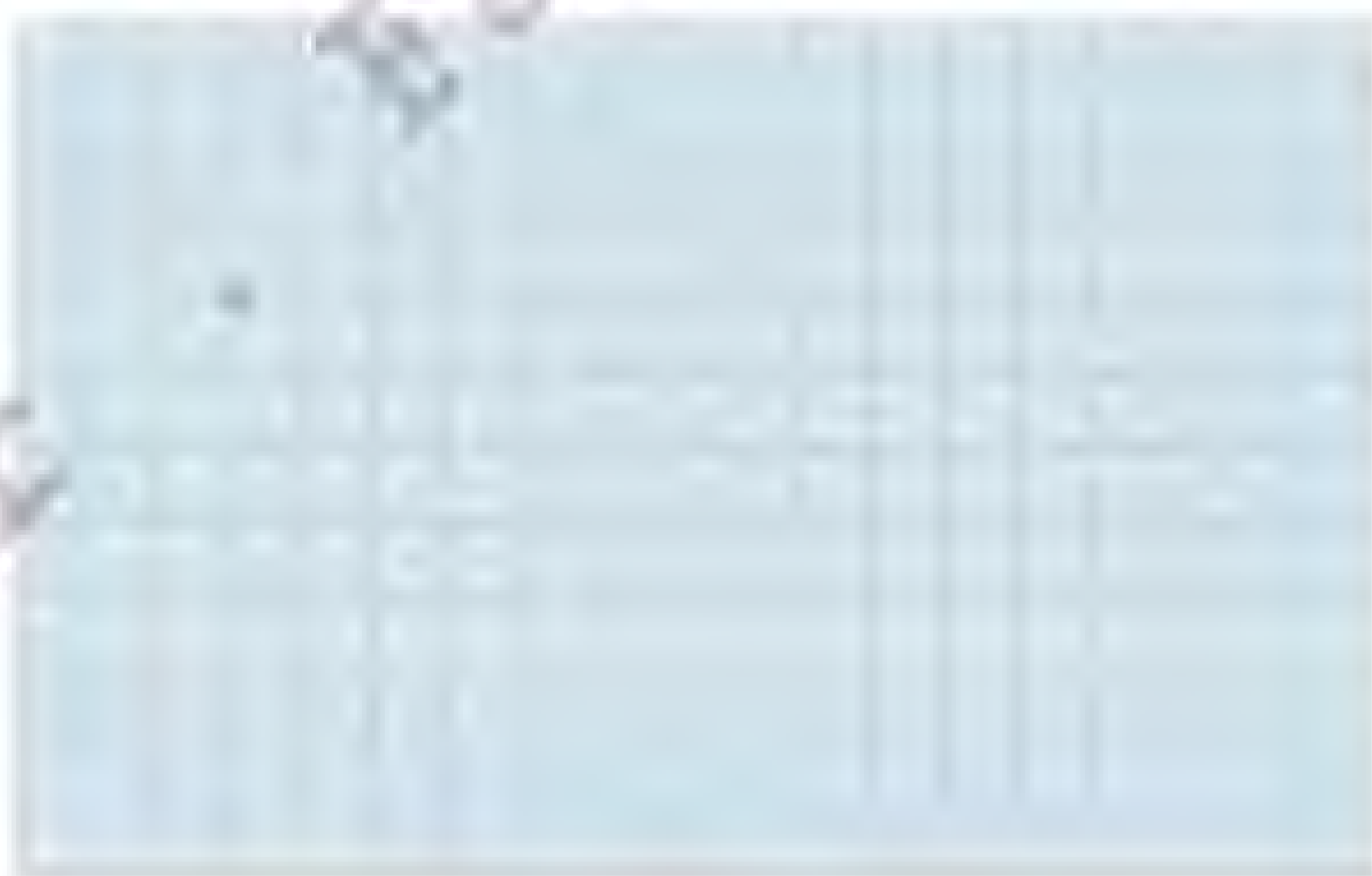
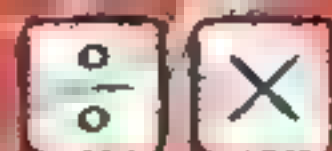
....

3

Draw triangle ABC where A(3, -1), B(5, 2), C(-2, 4), then draw its image under the rotation R(0, 180°).

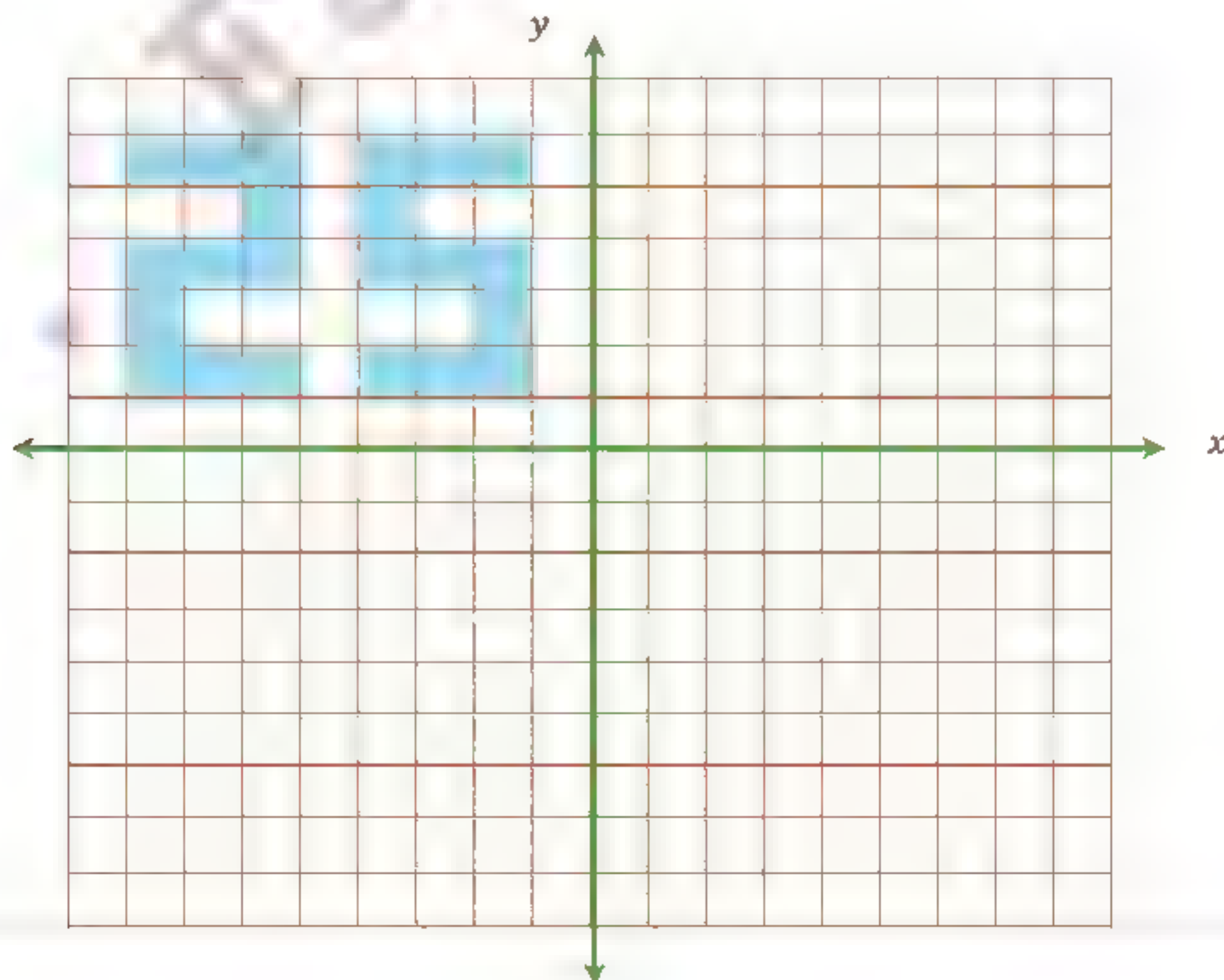
....

4

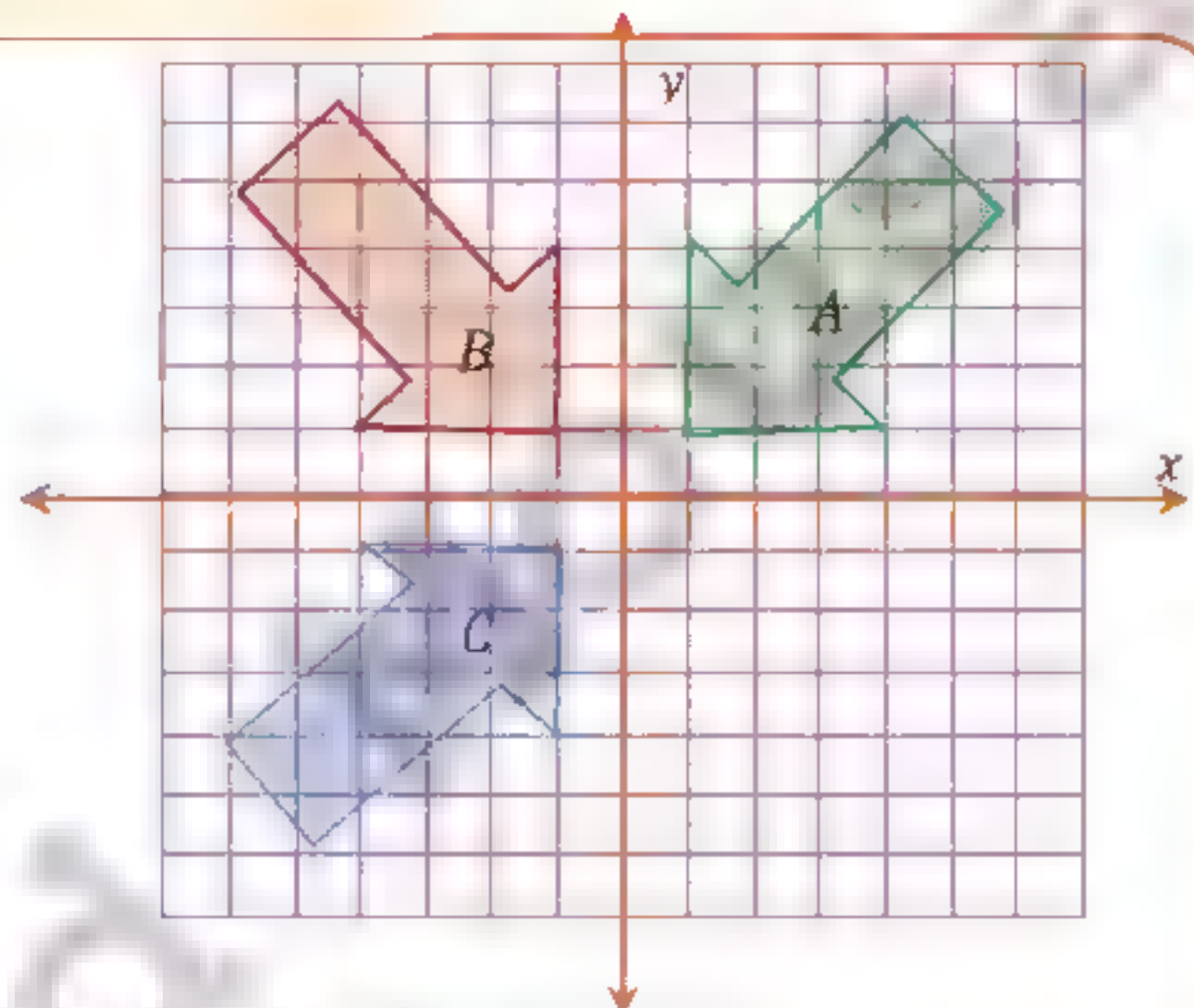
5

Draw triangle ABC where A(2, 0), B(4, 1), C(-1, 3), then draw its image under the rotation R(0, -180°)

....

6

In the following diagram, if the image of the number [illegible] under reflection over the [illegible]-axis is A, and its image under reflection over the X-axis is B, what is the value of $\sqrt[3]{B - A}$?

Unit 4 Lesson 4 | Composition of Geometric Transformations

learn

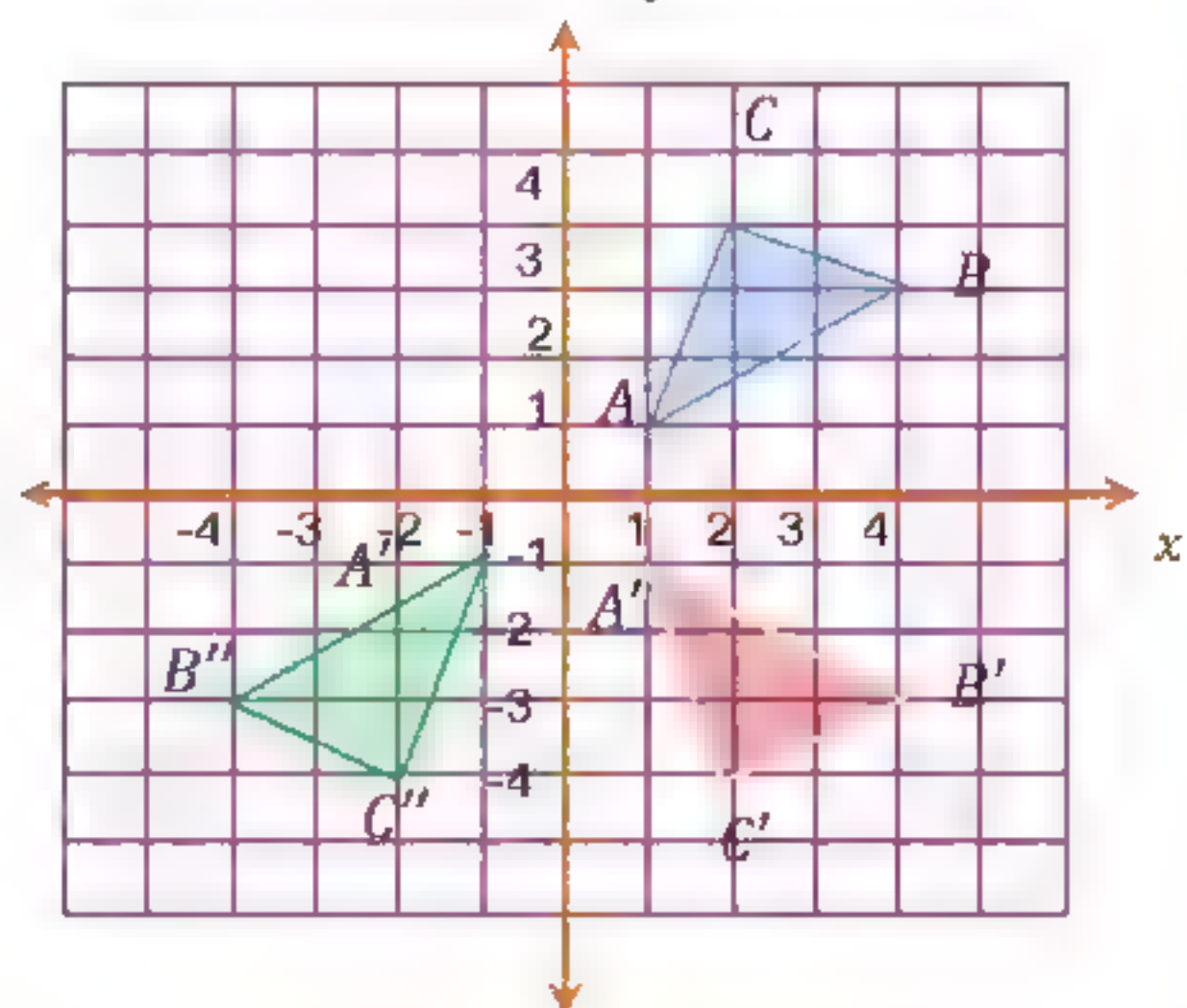
Composition of geometric transformations involves performing sequential geometric transformations on a geometric shape.
Sometimes, the resulting shape from the composition can be described by a single equivalent geometric transformation.
For example, arrow C is the image of arrow A by a reflection over axis l, followed by a reflection over the X-axis.

Example (1)

Draw ΔA'B'C', the image of ΔABC, by reflecting it over the X-axis, then draw ΔA''B''C'', the image of ΔA'B'C', by reflecting it over the Y-axis.

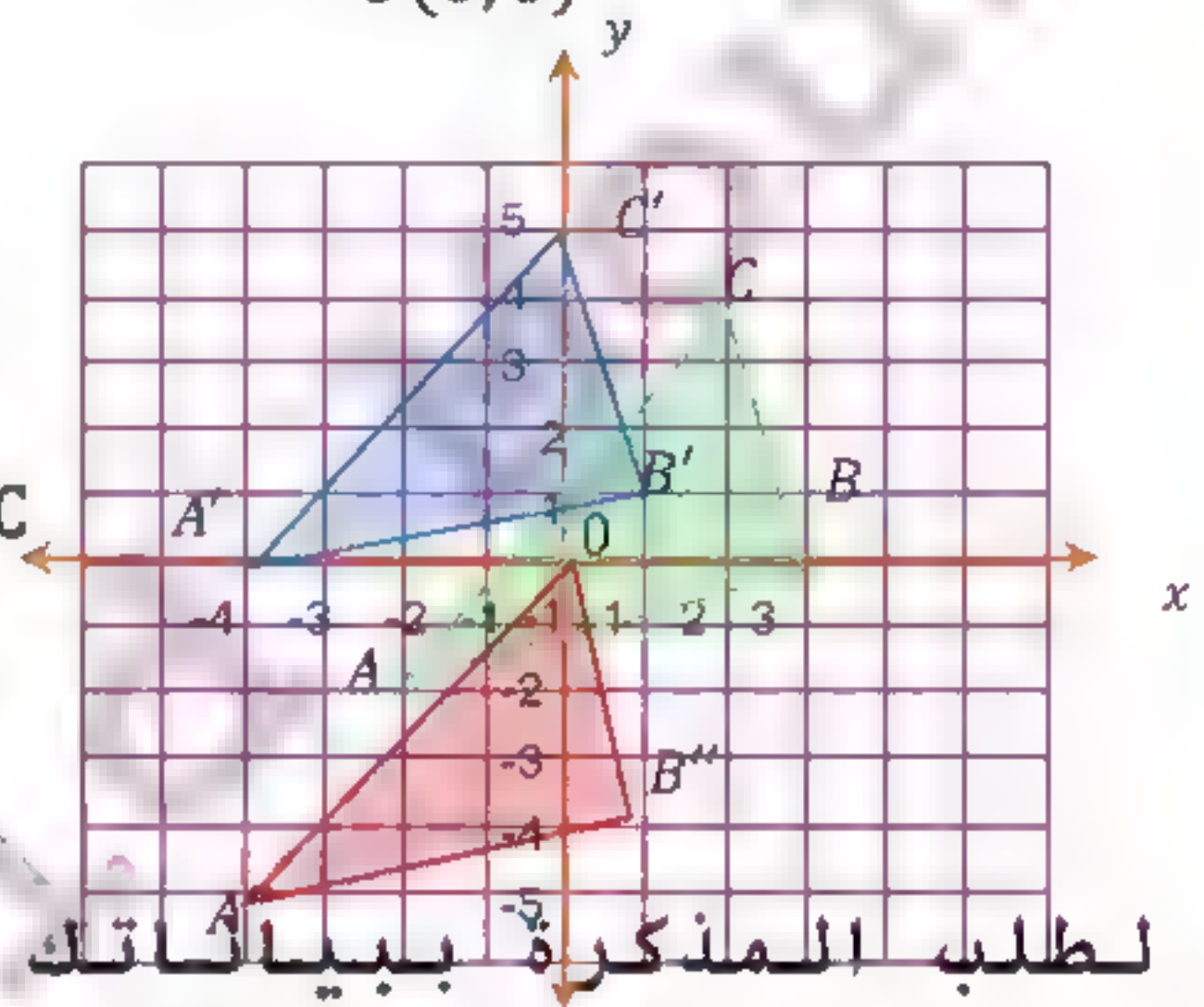
$A(1,1) \xrightarrow{\text{Reflection in the X-axis}} A'(1,-1) \xrightarrow{\text{Reflection in the Y-axis}} A''(-1,-1)$

$B(4,3) \xrightarrow{\text{Reflection in the X-axis}} B'(4,-3) \xrightarrow{\text{Reflection in the Y-axis}} B''(-4,-3)$

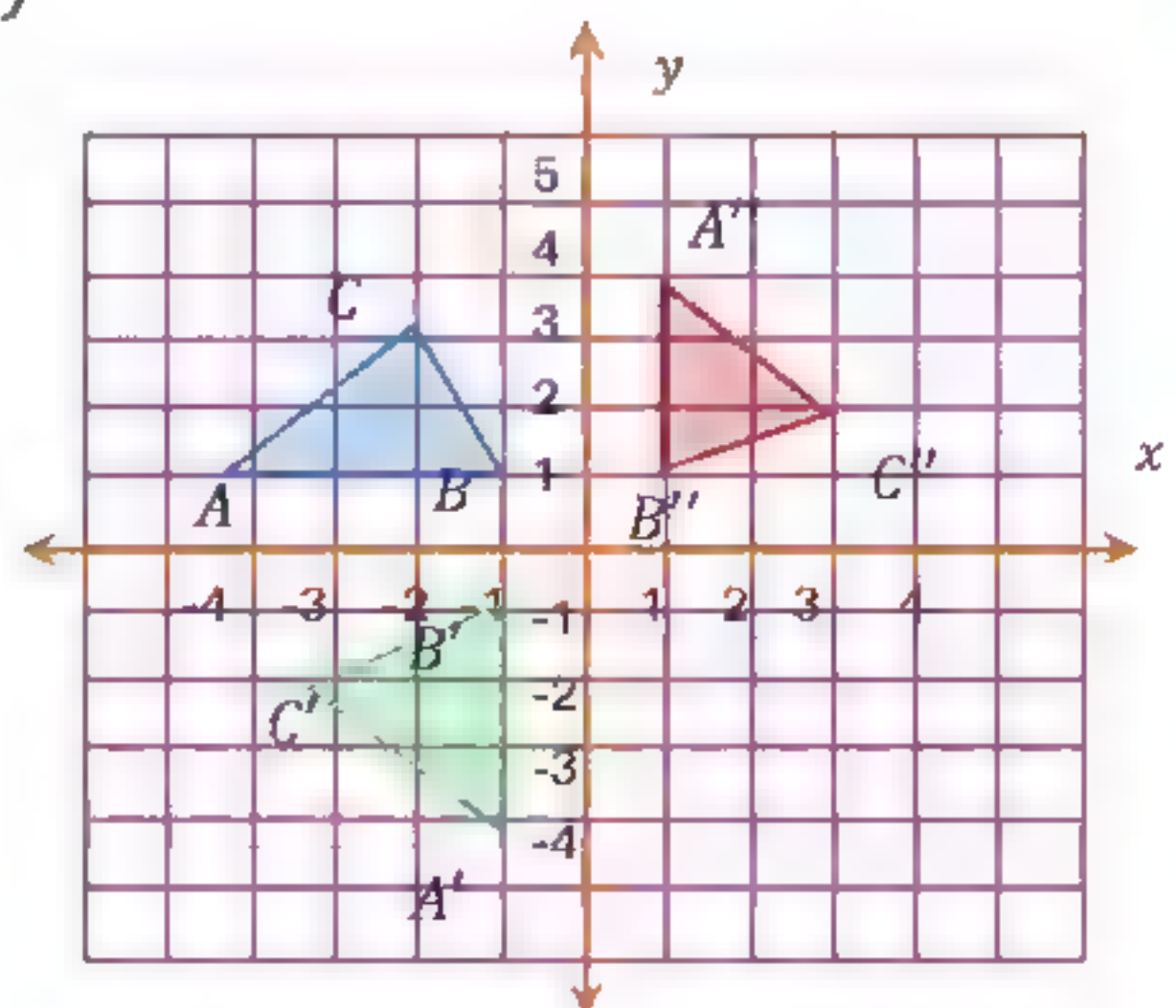
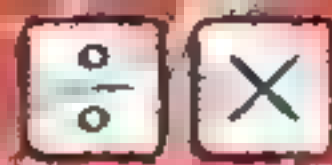
$C(2,4) \xrightarrow{\text{Reflection in the X-axis}} C'(2,-4) \xrightarrow{\text{Reflection in the Y-axis}} C''(-2,-4)$

The triangle ΔA''B''C'' is the image of ΔABC after reflection over the X-axis, followed by reflection over the Y-axis.

Draw ΔABC with points C(2, 4), B(3, 0), and A(-2, -1). Then draw its image after a translation of (-2, 1), followed by a translation of (0, -5).

$$A(-2,-1)\xrightarrow{Translation\ (-2,1)}A'(-4,0)\xrightarrow{Translation\ (0,-5)}A''(-4,-5)$$
$$B(3,0)\xrightarrow{Translation\ (-2,1)}B'(1,1)\xrightarrow{Translation\ (0,-5)}B''(1,-4)$$
$$C(2,4)\xrightarrow{Translation\ (-2,1)}C'(0,5)\xrightarrow{Translation\ (0,-5)}O(0,0)$$

The triangle ΔA"B"C" is the image of ΔABC after translation (-2, 1), followed by translation (0, -5).

Draw ΔABC with points C(-2, 3), B(-1, 1), and A(-4, 1). Then draw its image after a rotation R(0, 90°), followed by a rotation R(0, 180°).

$$A(-4,1)\xrightarrow{R(0,90^\circ)}A'(-1,-4)\xrightarrow{R(0,180^\circ)}A''(1,4)$$
$$B(-1,1)\xrightarrow{R(0,90^\circ)}B'(-1,-1)\xrightarrow{R(0,180^\circ)}B''(1,1)$$
$$C(-2,3)\xrightarrow{R(0,90^\circ)}C'(-3,-2)\xrightarrow{R(0,180^\circ)}C''(3,2)$$

The triangle ΔA"B"C" is the image of ΔABC after a rotation R(0, 90°), followed by a rotation R(0, 180°).

Draw ΔABC with points C(-2, 3), B(-1, 1), and A(-4, 1). Then draw its image after a translation of (2, 1), followed by a translation of (0, -2).

....

Draw ΔABC with points C(1, 1), B(0, 4), and A(4, 1). Then draw its image after a rotation R(0, 90°), followed by another rotation R(0, 90°).

....

Exercises (6)

Choose the correct answer from the given options:

1 What geometric transformation is equivalent to a reflection over the X-axis followed by a reflection over the Y-axis?

(a) Rotation R(0, 90°) | (b) Rotation R(0, 180°)

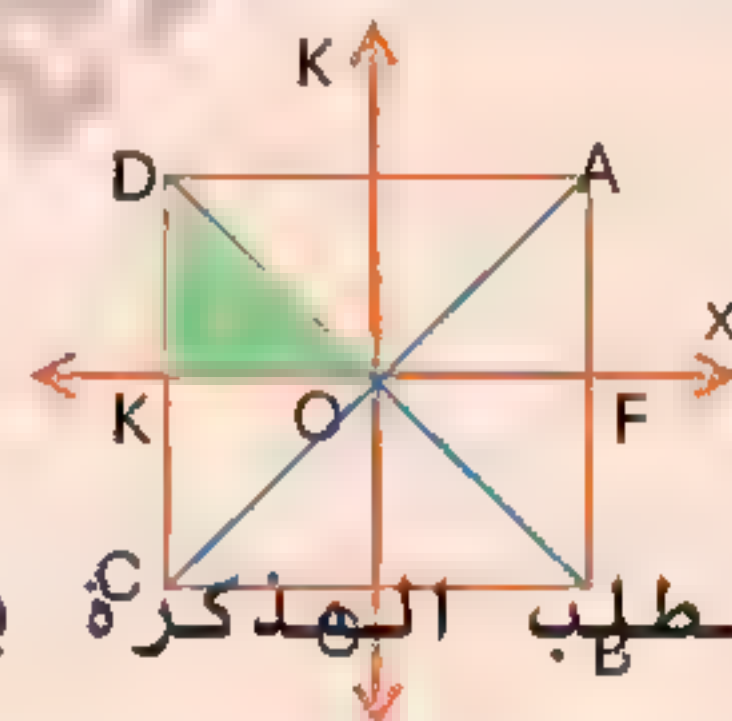
(c) Rotation R(0, 360°) | (d) Rotation R(0, 270°)

2 What is the geometric transformation equivalent to a translation of (1, 3)?

(a) Translation (1, 5) | (b) Translation (-1, -1)

(c) Translation (1, 1) | (d) Translation (0, 5)

3 In the given diagram, what is the image of ΔDKO under a reflection over the X-axis followed by a reflection over the Y-axis?

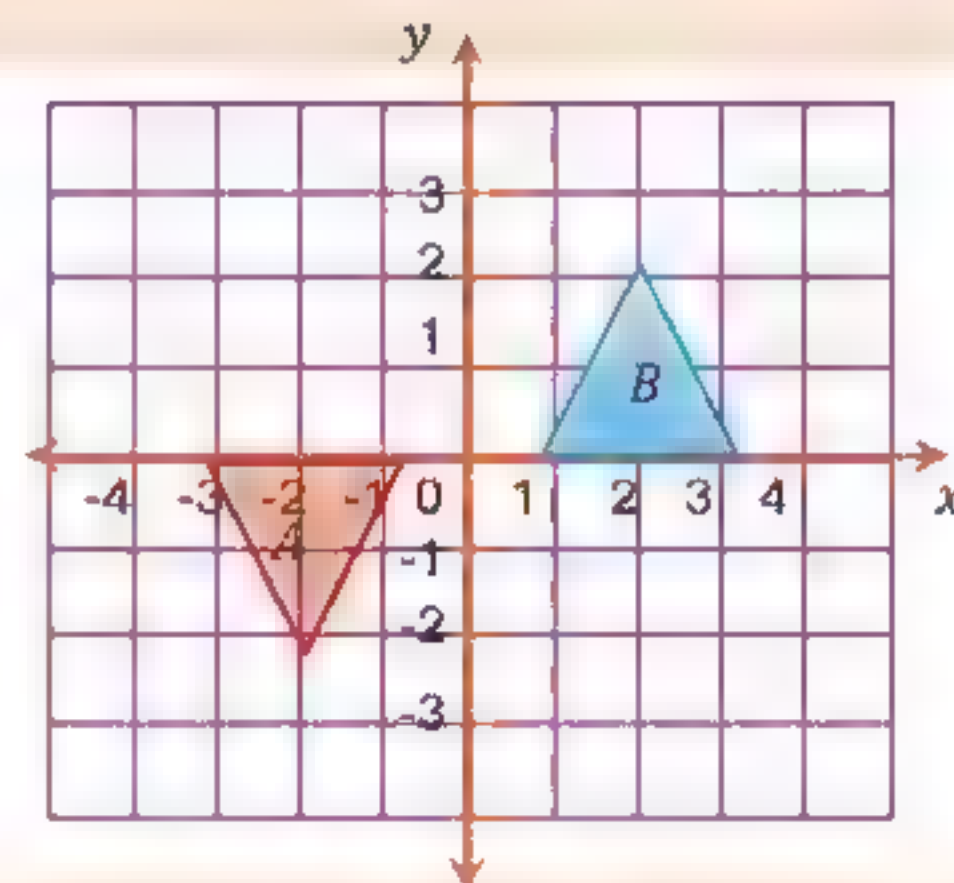
.

(a) ΔBFO | (b) Δ CKO | (c) Δ AFO | (d) Δ BGO

4 In the given diagram, which of the following transformations makes triangle B the image of triangle A?

(a) Translation by 4 units to the right followed by 2 units upward

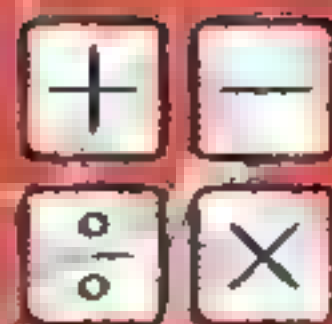
(b) Reflection over the Y-axis followed by a reflection over the X-axis

(c) Rotation R(0, 180°) followed by another rotation R(0, 180°)

(د) Rotation R(0, 90°) followed by a rotation R(0, 180°)

5 What is the image of the point (2, -3) under a reflection over the X-axis followed by a reflection over the Y-axis?

(a) (2 , 3) | (b) (-2 , -3) | (c) (−2 , 3) | (d) (3 , 2)

6 What is the image of the point (-3, 5) under a reflection over the X-axis followed by a reflection over the Y-axis?

(a) (3 , -5) | (b) (-3 , -5) | (c) (−3 , 5) | (d) (3 , 5)

7 What is the image of the point (-2, 4) under a rotation R(0, 90°) followed by a rotation R(0, 180°)?

(a) (-4 , 2) (b) (4 , 2) (c) (−4 , −2) (d) (4 , -2)

8 What is the image of the point (-1, 0) under a translation (1, 0) followed by a translation (2, -3)?

(a) (2 , -3) (b) (0 , 0) (c) (1, 0) (d) (-1 , 0)

9 What is the image of the point (-3, 0) under a rotation R(0, 90°) followed by a rotation R(0, -90°)?

(a) (3 , 0) (b) (0 , 3) (c) (0, −3) (d) (-3 , 0)

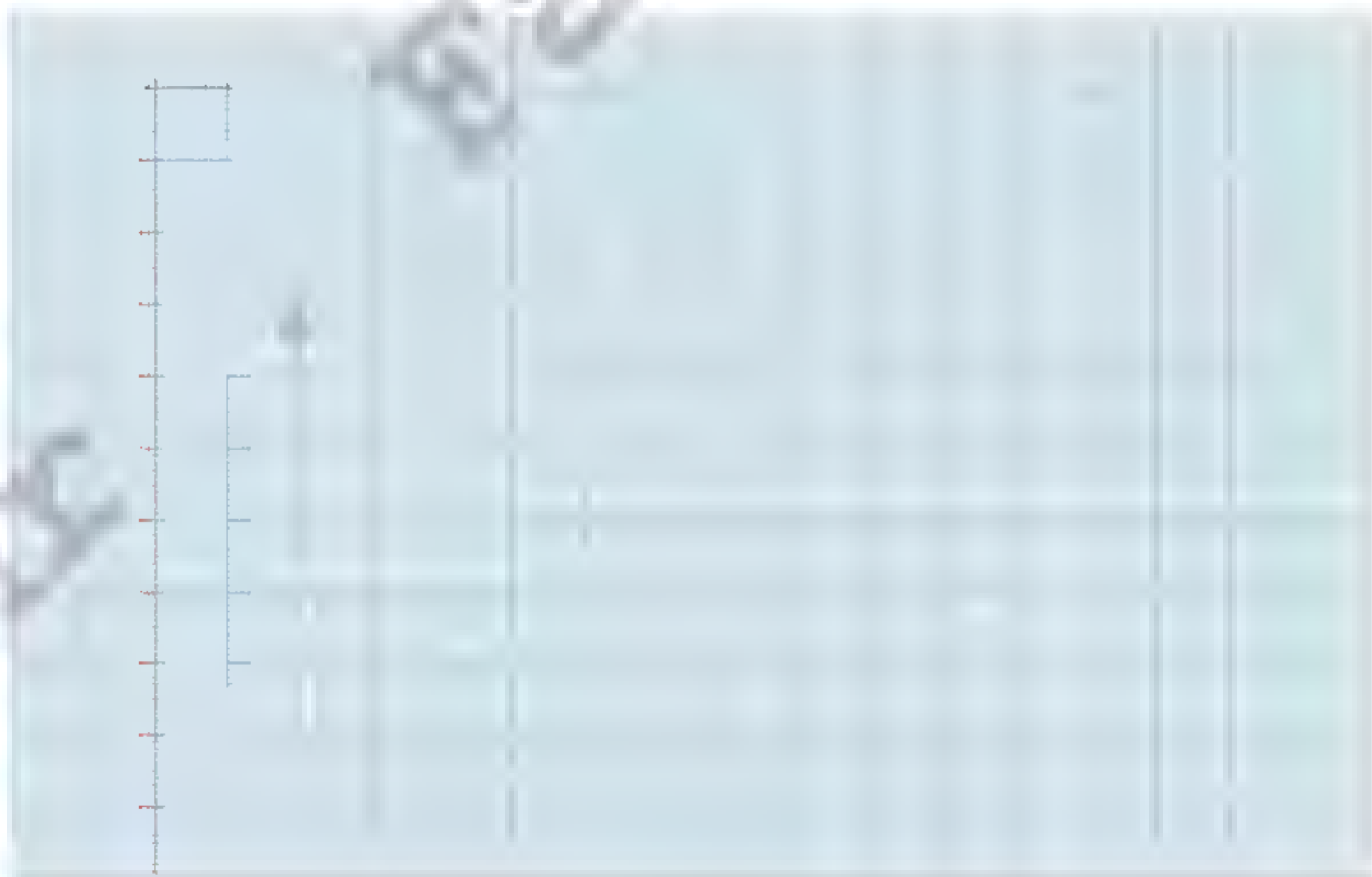
10 What is the image of the point (-2, 3) under a translation (x, y) → (x + 1, y - 2) followed by a translation (-1, 2)?

(a) (-3 , 5) (b) (0 , 0) (c) (−4, 3) (d) (-2 , 3)

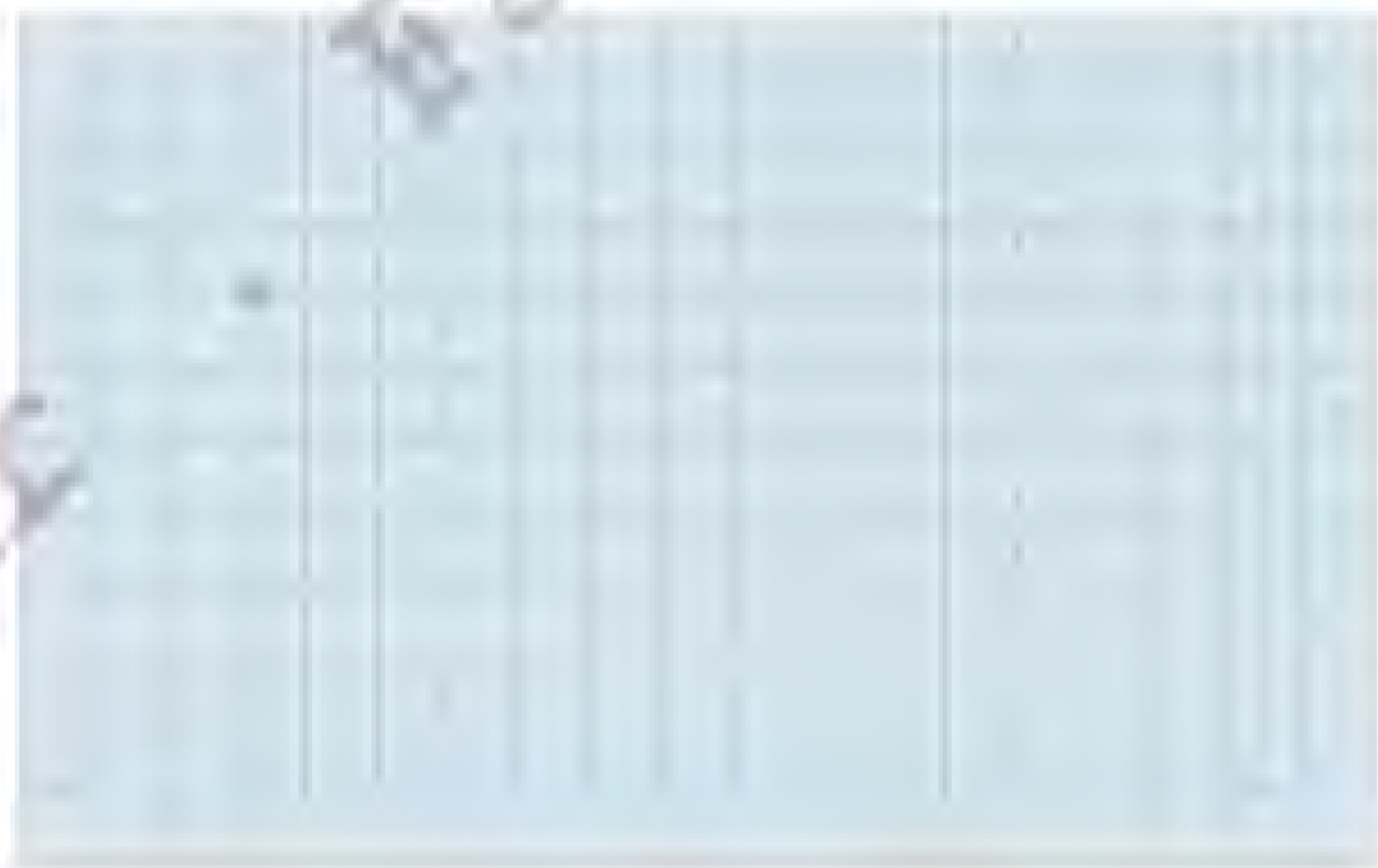
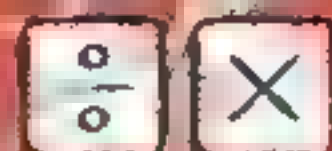
Example 2:

Draw triangle ABC with points C(2, 2), B(2, 0), and A(-1, 2) by reflecting it over the X-axis, followed by reflecting it over the Y-axis.

....

....

Draw quadrilateral ABCD with points C(3, -1), D(3, -3), B(0, -3), and A(-1, 1), then apply a translation of (-1, 0) followed by a translation of (-1, 5).

....

Draw triangle ABC with points C(-2, 3), B(-3, 1), and A(0, 0) by applying a rotation R(0, 90°), followed by a rotation R(0, -180°).

Plot triangle ABC on the coordinate plane with points C(3, 4), B(1, 4), and A(1, 0). Then draw its image by reflecting it over the X-axis, followed by reflecting it over the Y-axis.

Plot line segment AB where B(0, 3) and A(-1, 0) on the coordinate plane, then draw its image by applying a rotation R(0, 90°), followed by a rotation R(0, -180°).

Random Experiment
Sample Space - Events

Learn

Definition :

A sample is a small part from a large society that looks like this society and represents it syell and is selected randomly.

Types of samples

Samples are classified according to the way used in selecting its items, and inthis lesson , we introduce two types of samples:

(1) Systematic sample:

Systematic sample is the sample whose elements are selected from the elements of a society distributed randomly by following a certain system or method in selection.

(2) Random sample:

Random sample is the sample whose elements are selected from the elements of a society distributed randomly by following a random and irregular method or system of selecting.

- In this sample, each individual must get the samechance of selecting.

So, we can select its elements by two methods:

Manual method. Using the scientific calculator.

(3) Random Experiment:

A random experiment is any experiment where all possible outcomes can be determined before conducting it, but it is not possible to predict which specific outcome will actually occur when the experiment is performed .

(4) Sample Space (or Space of Outcomes):

The sample space is the set of all possible outcomes of a random experiment. It is usually denoted by the symbol S , and the number of elements in the sample space is represented by n(S) .

1. Experiment: Tossing a fair coin once and observing the visible side is a random experiment because:
   - You cannot determine the outcome until the experiment is performed.
   - You can predict all possible outcomes beforehand, which are: heads (H) or tails (T) .
   - The sample space is S = {T, H} .
   - The number of elements in the sample space is 2 , written as n(S) = 2 .
2. Experiment: Selecting a card with the letter "B" from a set of identical cards, all labeled with the letter "B," is not a random experiment because:
   - The result is certain and predictable before conducting the experiment—it will always be a card with the letter "B."
3. Experiment: Drawing a colored ball from a box containing identical balls of unknown colors is not a random experiment because:
   - It is impossible to predict the color of the ball or even determine the possible outcomes beforehand.

1. **Throwing a fair die once and observing the number on the upper face.**
   Type : Random experiment.
   S = {1 , 2 , 3 , 4 , 5 , 6} , n (S) = 6
2. **Drawing a ball from a bag containing a red ball, a blue ball, and a white ball, all identical, and observing its color.**

   ....
3. **Drawing a ball from a group of identical green balls and observing its color**

   ....
4. **Drawing a card from 7 identical cards numbered from 12 to 18 and observing the number on the card.**

   ....
5. **Drawing a ball numbered from a box containing identical balls with unknown numbers and observing the number on the ball.**

   ....

   ....

1. Drawing a ball from a bag containing a white ball, a yellow ball, a red ball, and a green ball, all identical, and observing its color.

   ....

   ....

2. Rolling a cube with faces numbered from 30 to 35 once and observing the number on the upper face.

   ....

   ....

3. Drawing a card from 7 identical cards, all labeled with the number 5, and observing the number on the card.

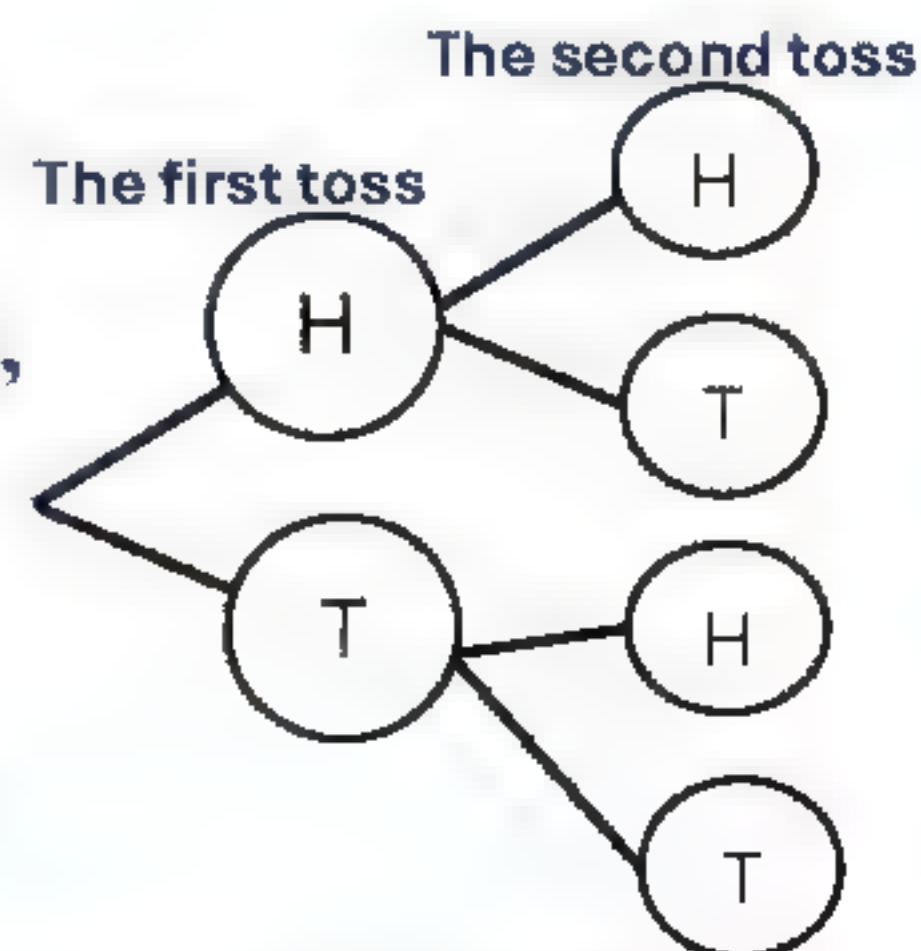
   ....

1. **Experiment: Tossing a fair coin twice in succession and observing the sequence of heads and tails.**

   - Each outcome of the experiment is an ordered pair, where the first element is the result of the first toss, and the second element is the result of the second toss.
   - The possible outcomes for each toss are: Head (H) or Tail (T).

     Using a tree diagram or systematic listing, the sample space is:

     S = {(H,H),(H,T),(T,H),(T,T)}
   - Number of elements: n(S) = 4

   Note: (T,H)≠(H,T)(T, H), as the order matters.

Experiment: Rolling a fair die twice in succession and observing the number on the upper face in each roll.

- Each outcome of the experiment is an ordered pair, where the first element is the result of the first roll, and the second element is the result of the second roll.
- The possible outcomes for each roll are {1,2,3,4,5,6 }.
- The sample space can be represented in tabular form or geometrically on a grid:

Tabular form: Rows represent the result of the first roll, and columns represent the result of the second roll.

slated version:

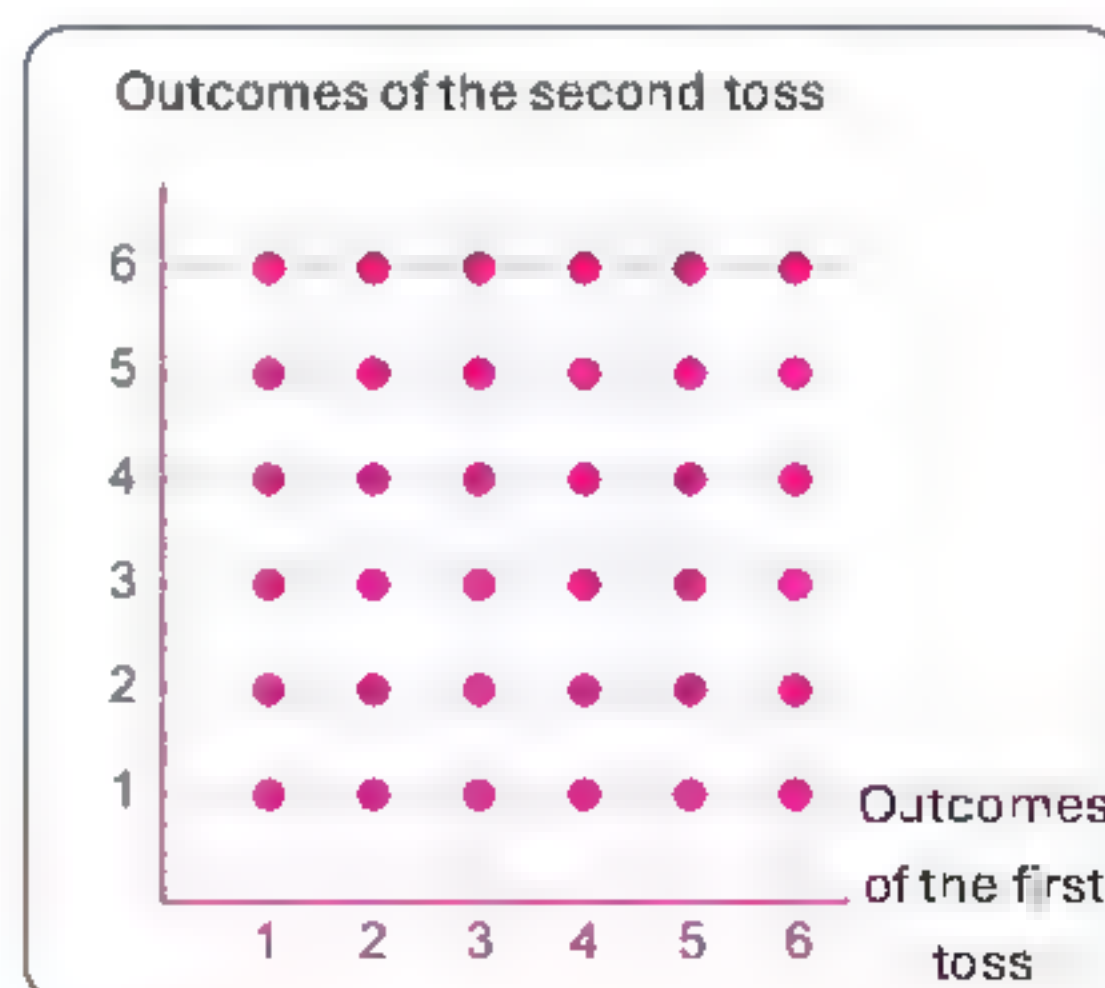
Second Roll / First Roll	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Each cell shows the pair (first roll, second roll). For example

Grid representation:

Each cell represents a unique outcome, such as (1,1),(1,2),...,(6,6).

- Total number of elements:

$n(S)=6\times6=36$

Learn

Event:

An event is a subset of the sample space.

Occurrence of an Event:

An event is said to occur if the outcome of the random experiment, after it is conducted, is one of the elements of the subset representing the event.

Certain Event (S):

A certain event is an event that must occur when the random experiment is conducted.

Impossible Event (ϕ):

An impossible event is an event that cannot occur under any circumstances when the random experiment is conducted.

Simple (or Elementary) Event:

A simple event is a subset of the sample space (S) that contains only one element.

Possible Event:

A possible event is a proper subset of the sample space (S).

Example

1

A fair die is rolled once:

- **What is the sample space for this random experiment?**
- **What are all the possible results if the goal is to observe an even number?**

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2

A single card is drawn from a set of cards numbered from 1 to 20:

- **What is the sample space for this experiment?**
- **Identify the results where the card corresponds to a prime number.**

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If a fair die is rolled once and the number appearing on the upper face is observed, write the sample space and determine each of the following events, specifying whether each event is simple, certain, or impossible:

1. Event (A) is the event of observing an even number.
2. Event (B) is the event of observing a number greater than 1.
3. Event (C) is the event of observing an even prime number.
4. Event (D) is the event of observing a number less than 7.

3

5. Event (E) is the event of observing the number 8.

The sample space is:

S={1,2,3,4,5,6}

1. A={2,4,6}
2. B={2,3,4,5,6}
3. C={2} (simple event)
4. D={1,2,3,4,5,6}= S (certain event)

(impossible event)

Form a two-digit number using the digits {7, 6, 4, 3} with different digits:

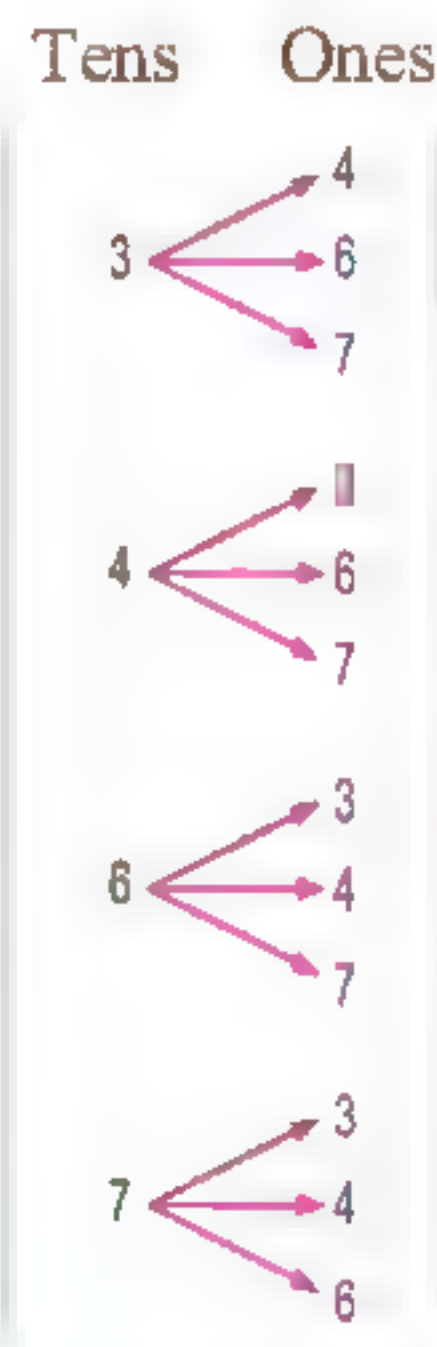
- Write the sample space for this experiment, then find the following events:

1. Event (A) is the event "the tens digit is odd."
2. Event (B) is the event "the number is divisible by 4."
3. Event (C) is the event "the sum of the two digits is 10."

4

Sample space:

S={34,36,37,43,46,47,63,64,67,73,74,76}

1. A={34,36,37,73,74,76}
2. B={36,64,76}
3. C={37,46,64,73}

In the experiment of selecting an integer from 2 to 11, write the sample space and determine the following events, specifying whether each is simple, certain, or impossible:

1. The event of observing an odd number.
2. The event of observing a number less than 16.
3. The event of observing a number less than or equal to 4.
4. The event of observing the number 6.
5. The event of observing an even number divisible by 9.
6. The event of observing a perfect square.

5

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In the experiment of rolling a fair die twice in succession and observing the number appearing on the upper face in both rolls, write the following events:

1. Event (A) is the event of obtaining two numbers whose sum is 8.
2. Event (B) is the event of obtaining two numbers, the larger of which is 3.
3. Event (C) is the event of obtaining two equal numbers.

6

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Exercises (1)

Question 1: Choose the correct answer from the given options:

1	Drawing a card from a set of identical cards numbered without knowing the numbers on them:						
(a)	Random experiment	(b)	Not a random experiment	(c)	Impossible event	(d)	Certain event
2	In the experiment of randomly selecting one digit from the number 5742, what is the sample space?						
(a)	{2 , 4 , 5}	(b)	{2 , 4 , 5 , 7}	(c)	{57 , 74 , 42}	(d)	{5742}
3	In the experiment of forming a two-digit number with different digits from the set {4, 3, 1}, how many elements are in the event representing "the resulting number is odd?"						
(a)	2	(b)	3	(c)	4	(d)	6
4	In the experiment of tossing a fair coin four times in succession, how many elements are in the sample space?						
(a)	2	(b)	4	(c)	8	(d)	16
5	In the experiment of rolling a fair die once, which of the following events is a simple event?						
(a)	The event of observing a number greater than 6	(b)	The event of observing an even prime number	(c)	The event of observing a number less than or equal to	(d)	The event of observing an odd prime number

Example 1 :

1. A coin is tossed twice:
   - Write the sample space for the random experiment.
   - Determine the event that represents "exactly one head appears."

   ....

   ....

2. You have a box containing 5 red balls and 4 blue balls, and one ball is drawn:
   - What is the sample space for this experiment?
   - Determine the event that represents drawing a blue ball

   ....

   ....

3. A random number is selected from the set {1,2,3, ... , 15 }:
   - Determine the event that represents selecting an odd number.
   - Determine the event that represents selecting a number divisible by 3.

4. You have a bag containing colored balls: 3 red, 2 blue, and 5 green, and one ball is chosen randomly:
   - Determine the event that represents selecting a ball that is not red.
   - Determine the event that represents selecting a green ball.

5. In the experiment of rolling a fair die once and observing the number appearing on the top face, write the sample space and then determine the following events, specifying whether each is simple, certain, or impossible:
   1. Event A: Observing a number greater than zero.
   2. Event B: Observing a number divisible by 3.
   3. Event C: Observing a number less than or equal to 4.
   4. Event D: Observing a number satisfying $5 > x$.
   5. Event E: Observing an odd number that is not prime.
   6. Event F: Observing a number greater than 4 and less than 5.
   7. Event G: Observing a number that is not a perfect square.

6. A bag contains 25 identical cards numbered from 1 to 25, and one card is drawn randomly, observing the number on it. Write the following events:
   1. Event A: Observing a number less than 4.
   2. Event B: Observing a multiple of 6.
   3. Event C: Observing an odd number divisible by 5.
   4. Event D: Observing a perfect cube.

7. A fair coin is tossed twice in succession, and the sequence of heads and tails is observed:

- Write the sample space (S) and describe the following events:

1. Event A: "Tail appears on the first toss."
2. Event B: "Tail appears in exactly one toss."
3. Event C: "The same side appears on both tosses."
4. Event D: "No heads appear."
5. Event E: "Different sides appear on both tosses."

8. A coin is tossed, and then a fair die is rolled, observing the top face of the coin and the number on the die:

- Represent the sample space using a tree diagram, and determine the following events:

1. Event A: "Tail and an even number appear."
2. Event B: "Head and an odd number appear."

9. In the experiment of rolling a fair die twice in succession, write the following events:

1. Event A: "The number 3 appears on the second roll."
2. Event B: "The sum of the two numbers is greater than or equal to 10."
3. Event C: "The sum of the two numbers is 15."

Sports: At a youth center offering activities during summer vacation, the sample space is: {Swimming, Squash, Volleyball, Tennis, Cycling, Football } .

- Event A: Choosing a ball game.
- Event B: Choosing a racket game.

10

A restaurant offers lunch options consisting of a main course and one type of appetizer:

- List the possible combinations for choosing a meal.

11

A spinning wheel with numbers 1 to 8 is spun:

- What is the sample space for this experiment?
- Determine the event that represents observing a number greater than 5.

12

Unit 4
Lesson 2

Theoretical Probability and Experimental Probability

Probability

Theoretical Probability:

Theoretical probability is based on the principle of equal chances or equal possibilities and is calculated as the ratio of the number of outcomes of the event to the total number of possible outcomes.

The probability

of any event occurrence $A \subset S$ is denoted by $P(A)$ and it is given by using the relation :

$$P(A) = \frac{\text{The number of elements of the event } (A)}{\text{The number of elements of sample space } (S)} = \frac{n(A)}{n(S)}$$

- **Remarks**
- **The impossible event** : is the event that has no chance for occurring
  the probability of the impossible event = Zero
- **The certain event** : is the event that has all the possible outcomes.
  the probability of the certain event = 1
- **The value of probability** of any event is notless, than zero and not more than one
  $0 \leq$ The probability of an event occurrence ≤ 1

Example 1:

When rolling a fair die once and observing the top face, find the probability of the following events:

(1)Event A: Getting an even number **(2)Event B:Getting a number less than 8**

(3)Event C: Getting an odd prime number **(4)Event D:Getting the number 4**

(5)Event E:Getting a number greater than 6

All possible outcomes are: 1, 2, 3, 4, 5, 6, and their total is 6.

Since the even numbers are 2, 4, 6, their total is 3:

1 $\therefore P(A) = \frac{3}{6} = \frac{1}{2}$, $P(A) = 50\%$ or $P(A) = 0.5$ or $P(A) = \frac{1}{2}$

Since all numbers are less than 8, their total is 6:

$\therefore P(B) = \frac{6}{6} = 1$, $P(B) = 100\%$ or $P(B) = 1$

Since the odd prime numbers are 3, 5, their total is 2:

$\therefore P(C) = \frac{2}{6} = \frac{1}{3}$, $P(C) = 33\frac{1}{3}\%$ or $P(C) = 0.\overline{3}$ or $P(C) = \frac{1}{3}$

The number 4 appears only once: $\therefore P(D) = \frac{1}{6}$

There is no number greater than 6, so: $\therefore P(E) = \frac{0}{6} = 0$

A card is randomly drawn from a set of identical cards numbered 5 to 14. Find the probability of the following events:

(1) Getting an odd number (2)Getting an even number greater than 9

(3)Getting a prime number (4)Getting a number less than 5

2 (5)Getting a perfect square

....

....

....

....

fair coin is tossed twice, and the sequence of heads (H) and tails (T) is observed. Find the probability of the following events:

(1) Event A: Getting two heads (2) Event B: Getting at least one head

(3)Event C: Getting the same result in both tosses

(4) Event D: Getting a head in the first toss

All possible outcomes are: (H, H), (H, T), (T, H), (T, T), and their total is 4.

outcomes with two heads are (H, H), and their total is 1.

3 $\therefore P(A) = \frac{1}{4}$

outcomes with at least one head are (H, H), (H, T), (T, H), and their total is 3.

$\therefore P(B) = \frac{3}{4}$

outcomes with the same result in both tosses are (H, H),(T, T) and their total is 2.

$\therefore P(C) = \frac{2}{4} = \frac{1}{2}$

outcomes with a head in the first toss are (H, T), (H, H), and their total is 2.

$\therefore P(D) = \frac{2}{4} = \frac{1}{2}$

A bag contains 1 red ball, 6 blue balls, and 3 green balls, all identical. If a ball is drawn randomly from the bag, find the probability of the following events:

(1)The ball is blue (2)The ball is white (3)The ball is red

(4)The ball is green (5) The ball is blue or green (6)The ball is not green

Assuming (Red = R) , (Blue = B) , (Green = G) , (White = W):

The total number of balls = 1 + 6 + 3 = 10 balls.

4

$(1) P(B) = \frac{6}{10} = 0.6$ (2) $P(W) = \frac{0}{10} = 0$

$(3) P(R) = \frac{1}{10} = 0.1$ (4) $P(G) = \frac{3}{10} = 0.3$

$(5) P(B \text{ or } G) = \frac{6+3}{10} = \frac{9}{10} = 0.9$ (6) $P(\text{Not } G) = \frac{1+6}{10} = \frac{7}{10} = 0.7$

Alternative solution for calculating the probability that the drawn ball is not green:

$\because P(G) = 0.3$, $\therefore P(\text{Not } G) = 1 - 0.3 = 0.7$

5. A cinema shows the following films:
3 comedy films, 2 animated films, 1 horror film, and 4 social films. If a film is chosen randomly, find the probability of the following events:

1. The film is social
2. The film is horror
3. The film is comedy or social
4. The film is not comedy

6. From the set of numbers {7, 5, 3, 2}, form a two-digit number with different digits. If one of these numbers is chosen randomly, find the probability of the following events:

1. The tens digit is greater than the units digit
2. The number is prime
3. One of the digits is even

Note that

- The sum of the probabilities of all possible outcomes of any random experiment equals 1.
- For any event A:

$$P(A) + P(\text{Not } A) = 1$$

Experimental Probability

It is based on conducting an experiment scientifically, recording its results, and then using these results to calculate the probability as follows:

$$\text{Experimental Probability of event (A)} \; \frac{\text{Number of times event (A) occurs}}{\text{Number of trials conducted}}$$

Example 1:

1

If a fair coin is tossed 100 times and heads appeared 41 times, find the experimental probability of:

1. Getting heads (H) , **The number of times heads appeared = 41**

$\therefore P(H) = \frac{41}{100} = 0.41 = 41\,\%$

1- Getting tails (T) , $100 - 41 = 59$

$\therefore P(T) = \frac{59}{100} = 0.59 = 59\,\%$

Note:

In the experiment of tossing a fair coin once, the theoretical probability of getting heads is $\frac{1}{2}$ = 50%. Therefore, there is a difference between the experimental probability of getting heads (41%) and the theoretical probability (50%). The more times the experiment is conducted, the closer the experimental probability will get to the theoretical probability.

2

A spinning wheel is divided into equal-colored sectors, and it is spun 50 times. The table shows the number oftimes the pointer stopped on each color.

Color	Number of Times
Red	8
Blue	9
Yellow	13
Green	9
Purple	11

1. **Find the experimental probability of the pointer stopping on yellow:**

   $P(\text{Yellow}) = \frac{13}{50} = 0.26 = 26\%$

2. **Find the theoretical probability of the pointer stopping on yellow:**

   Since there are five equally distributed colors on the spinning wheel, the theoretical probability is

   $P(\text{Yellow}) = \frac{1}{5} = 0.2 = 20\%$

3. **if the number of spins increases to 500, what is expected about the chance of the pointer stopping on yellow?**

   As the number of spins increases, it is expected that the experimental probability of the pointer stopping on yellow will decrease and get closer to the theoretical probability of 20%.

A fair die was rolled 150 times, and the observed outcomes are as follows:

1. Find the experimental probability of getting the number 2.
2. Find the experimental probability of not getting the number 5.
3. Find the theoretical probability of getting the number 2.

Number	times
1	28
2	19
3	23
4	28
5	25
6	27

3 ....
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When a colored ball is drawn from a bag containing four identical balls (red, green, blue, and white), the percentages of draws for each color are as follows:

1. Find the experimental probability of not drawing a blue ball.
2. If the experiment was conducted 200 times, find the number of times a white ball was drawn.
3. Find the theoretical probability of drawing a red ball.

the color	percentage of times
Green	[illegible]
Blue	27%
White	28%
Red	22%

4 ....
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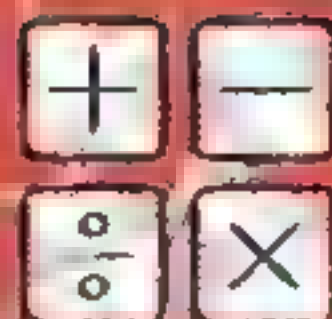

Exercises (2)

Question 1: Choose the correct answer from the given options:

1 If you are considering buying one pen from a collection of identical pens containing 5 red pens, 2 blue pens, and 3 black pens, and you choose a pen randomly, what is the probability that the pen is blue?

(a)	$\frac{1}{4}$	(b)	$\frac{1}{5}$	(c)	$\frac{2}{15}$	(d)	$\frac{1}{15}$

2 In the experiment of rolling a fair die once, what is the probability of getting a number divisible by 2 ?

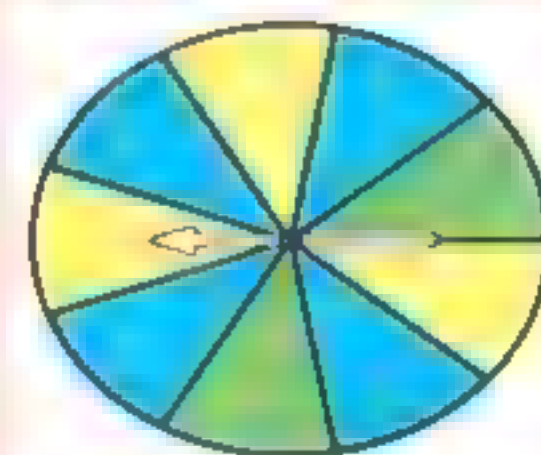
(a)	75%	(b)	50%	(c)	$33\frac{1}{3}\%$	(d)	0

3 When rolling a fair die 10 consecutive times, if the number 4 appears twice on the upper face of the die, what is the experimental probability of not getting the number 4 ?

(a)	$\frac{8}{10}$	(b)	$\frac{5}{6}$	(c)	$\frac{2}{10}$	(d)	$\frac{1}{6}$

4 If A is an event from a random experiment with equal chances, and the probability of event A is 40%, while the sample space has 15 elements, how many elements does event A have?

(a)	10	(b)	6	(c)	4	(d)	2

5 Hamza has a spinner divided into 9 equal sections, as shown in the accompanying diagram. When spun, the pointer lands randomly on one of the sections. What is the probability that the pointer lands on blue or yellow?

(a)	$\frac{8}{9}$	(b)	$\frac{7}{9}$	(c)	$\frac{4}{9}$	(d)	$\frac{2}{9}$

Example 2:

1 When rolling a die and observing the upper face, complete the following:

1. Probability of getting a number greater than 2: ……………………..
2. Probability of getting a number less than 3: ……………………..
3. Probability of getting an even number: ……………………..
4. Probability of getting the number 4: ……………………..
5. Probability of getting the number 7: ……………………..
6. Probability of getting a number less than or equal to 6: ……………………..
7. Probability of getting a prime number: ……………………..
8. Probability of getting an even prime number: ……………………..

2	The probability of an impossible event = …………… The probability of a certain event = ……………
3	If a coin is flipped once, the probability of getting heads = ……………
4	If 10 cards are numbered from 1 to 10 and one card is drawn at random, the probability that the card has an odd number = …………
5	In the experiment of rolling a die once and observing the upper face, the probability of getting a number less than 1 = ……………
6	A box contains 48 oranges, 4 of which are spoiled. If one orange is drawn randomly, the probability of it being spoiled…………… =
7	An activity room contains 3 doors numbered 1 to 3. If a student exits through one of the doors, the probability of exiting through door number 2 = …………
8	If the probability of a certain event occurring is $\frac{5}{8}$, the probability of the event not occurring = ………
9	If the probability of a person in a city of 200,000 people having a certain disease is 0.003, the expected number of people with the disease in this city = ………. people.
10	A bag contains 15 balls, 9 of which are red. If a ball is chosen randomly, what is the probability that it is not red? ……………
11	In the experiment of flipping a coin, if the probability of getting heads is 0.6, what is the probability of not getting heads? …………
12	In a competition with 10 questions, if the probability of solving a random question correctly is 0.8, what is the probability of solving it incorrectly? ……………
13	If the probability of a certain event occurring is $\frac{2}{7}$, what is the probability of it not occurring? ………
14	In the experiment of rolling a die, if the probability of getting an odd number is 0.5, what is the probability of getting an even number? …………
15	In a race with 5 contestants, if the probability of a certain contestant winning is 0.25, what is the probability of them losing? ………
16	If the probability of drawing a prime-numbered card from a set of cards numbered 1 to 10 is 0.4, what is the probability of drawing a non-prime card ? ………………
17	A bag contains 12 balls: 7 blue balls and 5 green balls. If a ball is chosen randomly, what is the probability that it is not blue……………… ?
18	In a product testing experiment, if the probability of the product being defective is 0.1, what is the probability of the product being non-defective? …………
19	If the probability of a student passing an exam is $\frac{3}{5}$, what is the probability of failing?…

Example 3:

1	When a card is drawn from a bag containing 25 cards numbered from 1 to 25, calculate the probability that the card has: 1. A number divisible by 5: …………………… 2. A number greater than or equal to 20: …………………… 3. A perfect square: ………………
2	When a card is chosen randomly from 8 cards numbered 1 to 8: Write the sample space and calculate the probability of the following events: 1. Getting an even number: …………………… 2. Getting an odd number: …………………… 3. Getting a number greater than or equal to 6: …………………… 4. Getting a number divisible by 3: ………………
3	When a letter is chosen randomly from the word "SAMEH," calculate the probability of choosing: 1. The letter S: …………………… 2. The letter E: …………………… 3. The letter R: ………………
4	A bag contains 5 red balls, 3 yellow balls, and 2 black balls. If a ball is chosen randomly, calculate: 1. The probability that the ball is yellow: …… 2. The probability that the ball is yellow or red: …………………… 3. The probability that the ball is not yellow: ………………
5	If a card is chosen randomly from 10 cards numbered 1 to 10, what is the probability that the card shows: 1. An odd number: …………………… 2. A prime number: …………………… 3. An even number: …………………… 4. An odd number greater than 3: …………
6	If a die is rolled once, what is the probability of the following events: 1. Getting an even number less than or equal to 4: …………………… 2. Getting a number between 0 and 10: …………………… 3. Getting a number divisible by 7: …………………… 4. Getting a number not divisible by 2: ……………
7	A cube is designed so that each pair of opposite faces carries one of the numbers 1, 2, and 3. If the cube is rolled and the upper face is observed: 1. Write the sample space: …………………… 2. What is the probability that the number on the upper face is 2? …………………… 3. What is the probability that the number on the upper face is odd? ………………

8 A classroom has 40 students, 30 of whom passed mathematics, 24 passed science, and 20 passed both. If a student is chosen randomly:

1. Calculate the probability that the student passed mathematics: ……………..
2. Calculate the probability that the student failed science: …………………..
3. Calculate the probability that the student failed both mathematics and science: ……………………………………………………………………………………

9 A bag contains 30 identical balls. Hany picked a ball randomly and found it to be red. If the probability of picking a red ball is $\frac{2}{5}$, calculate the number of red balls in the bag:

……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………

10 A box contains 80 identical balls, some of which are red, and the rest are blue. If the probability of picking a red ball is $\frac{1}{4}$, calculate the number of blue balls:

……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………

11 A bag contains a certain number of identical balls, 5 of which are white and the rest are red. If the probability of picking a red ball is $\frac{2}{3}$, find the total number of balls:

……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………

Example 4:

1 **A classroom has 15 students, 4 with black hair, 5 with brown hair, and 6 with blonde hair. If a student is chosen randomly, find the probability that the student:**

1. **Has black hair**
2. **Does not have brown hair**
3. **Has blonde or brown hair**

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……………………………………………………………………………………
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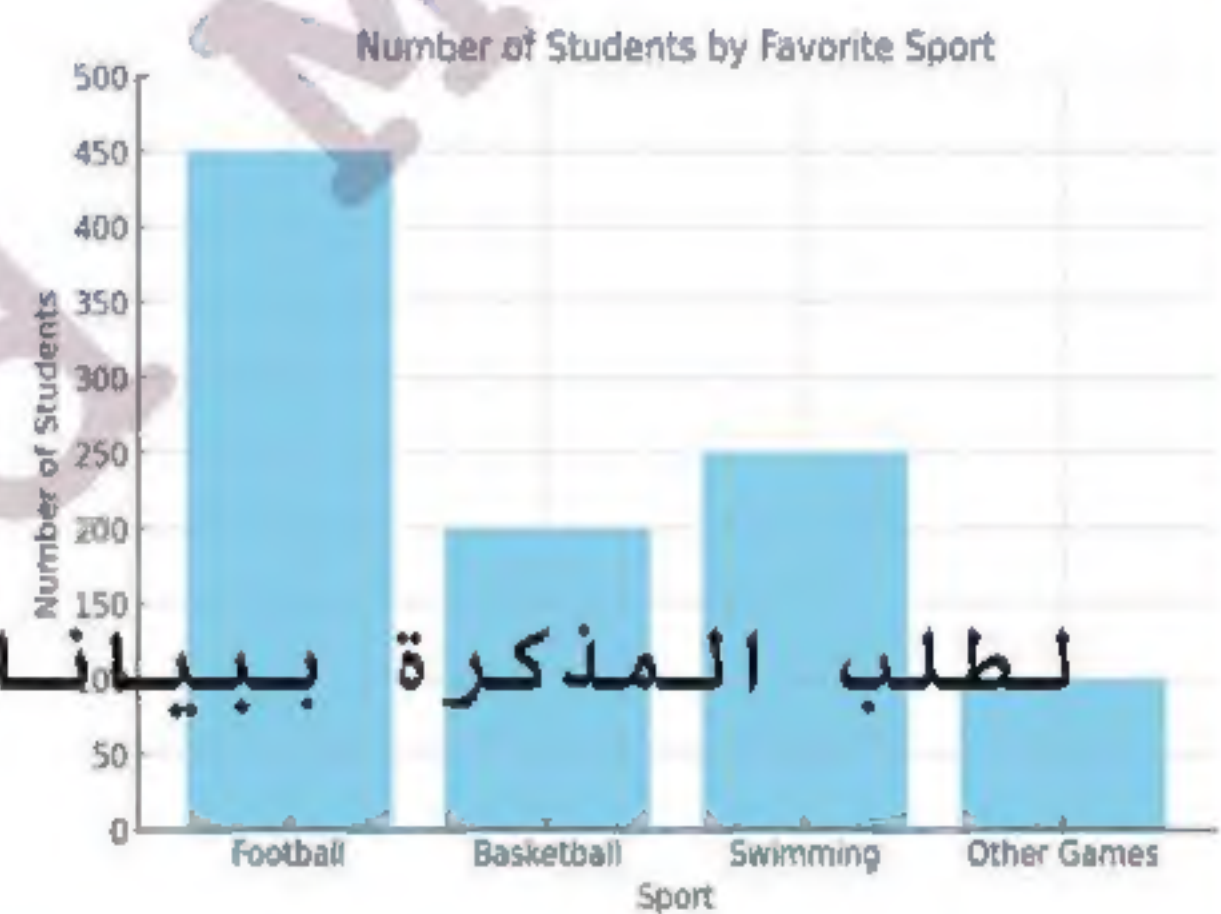
2	In the experiment of rolling a fair die once, find the probability of the following events: 1. Event A: Getting an odd number 2. Event B: Getting a number greater than 4 3. Event C: Getting the number 3 4. Event D: Getting a number less than 7 5. Event E: Getting a number satisfying the inequality "$x \geq 2$" 6. Event F: Getting a factor of the number 6 ....
3	If a card is drawn randomly from identical cards numbered 20 to 29, find the probability that the card: 1. Has a number greater than 25 2. Has a number less than 20 3. Has a prime number 4. Has an even number ....
4	If one number is chosen randomly from the set {13, 17, 19, 23, 29, 31}, find the probability that the sum of the digits of the chosen number is an even number. ....

5	In the experiment of forming a two-digit number using the set of digits {7, 5, 4}, find the probability of the following events: 1. Event A: The sum of the digits is 9 2. Event B: The tens digit is odd 3. Event C: The product of the digits is 35 4. Event D: The tens digit equals the units digit ....
6	The bar graph shows the favorite sports of 1000 students. If a student is chosen randomly: 1. What is the probability that they prefer basketball? 2. What is the probability that they do not prefer swimming? 3. What is the probability that they prefer football? ....
7	The stem-and-leaf plot shows the number of hours 30 students spend studying weekly. If one student is chosen randomly, find the probability that the student: 1. Studies more than 32 hours per week 2. Studies less than 27 hours per week 3. Studies more than 16 hours or less than 30 hours per week ....

Stem	Leaves							
0	1	4	5	6	8	8	9	
1	0	0	1	1	1	6	7	8
2	0	1	2	2	3	4	5	5
3	0	1	3	4	5	6	6	

Key: 1 | 6 means 16